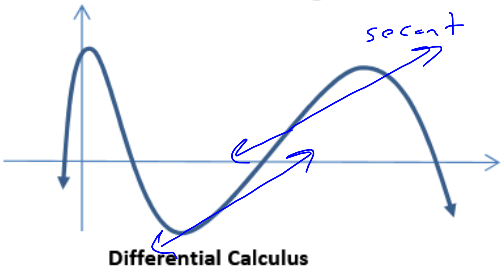
MCV4U | **Limits** | Unit 3, Lesson 1

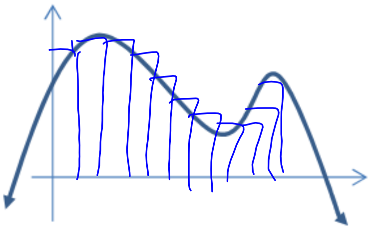
Calculus: The Two Big Questions



Differential Calculus

Average Rate of Change: slope of secant

Instantaneous Rate of Change: slope of tangent



Integral Calculus

Area under a curve

This course deals primarily with differential calculus.

Both Calculus questions require some knowledge of the concept of a ***limit.*** We will look at ***limits*** today.

The term limit asks us to find a value that is approached by *f(x)* as *x* approaches ***a*** but is not equal ***a***. This value is written as $\boxed{\lim_{x \to a} f(x)}$.

To consider this limit x must be defined at all points in some interval around ***a*** but not necessarily at x = a.

Consider the example below. Note that x cannot equal zero. However through some trial and error we can see that as x gets closer to zero, the value of the expression approaches some number. What is it?

$$\lim_{x \to 0} \frac{\sqrt{1+4x} - \sqrt{1-x}}{x}$$

$= 2?$

Consider the example above again. We can further classify limits as right or left sided limits using the following notation.

$$\lim_{x\to 0^-} \frac{\sqrt{1+4x}-\sqrt{1-x}}{x}$$

left-sided limit

$$\lim_{x\to 0^+} \frac{\sqrt{1+4x}-\sqrt{1-x}}{x}$$

right-sided limit

Notice that in the example above that our limit does not change if we approach from the left or right side. In order for a limit to exist, the following 3 conditions must be met:

$\lim\limits_{x\to a^-} f(x)$ must exist,

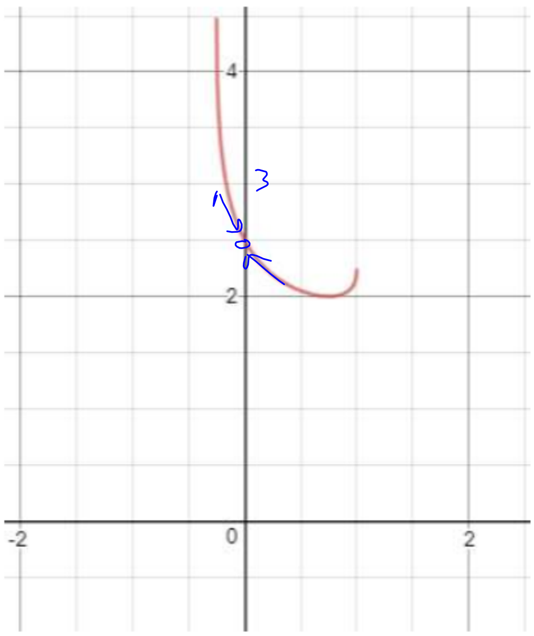
$\lim\limits_{x\to a^+} f(x)$ must exist, and

$\lim\limits_{x\to a^-} f(x) = \lim\limits_{x\to a^+} f(x)$

If the above three conditions do hold we say the limit exists and would just write $\lim\limits_{x\to a} f(x)$ (not specifying the right or left side).

Consider the graph of $f(x) = \frac{\sqrt{1+4x}-\sqrt{1-x}}{x}$.

There is a hole in the graph at x=0 (since x cannot be in the domain of *f(x)*).



How do we evaluate limits algebraically? (not using trial and error).

Some limits can be done very easily, simply by substitution. All polynomials and with some other functions this is possible.

Examples

$\lim\limits_{x\to 2}(x^2-3)$

$= 2^2-3$

$= 1$

$\lim\limits_{h\to 4}\frac{5}{h+2}$

$= \frac{5}{4+2} = 5/6$

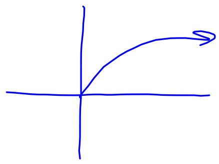
Some limits may not exist and it is possible to show this quite easily.

Examples

$\lim\limits_{x\to 0}\frac{1}{x}$

$\lim\limits_{x\to 0^-}\left(\frac{1}{x}\right) = -\infty$ $\quad$ $\lim\limits_{x\to 0^+}\left(\frac{1}{x}\right) = +\infty$

$\lim\limits_{x\to 0}\sqrt{x}$



$\lim\limits_{x\to 0^-}\sqrt{x}$ does not exist

The most interesting limits (like the first example today) are those that have an ***indeterminate form***. These limits are characterized by an inability to substitute the value, as we get a $\frac{0}{0}$. They require a different strategy.

Examples

$\lim\limits_{x\to 3}\frac{x^3-x^2-6x}{(x-3)}$

$= \lim\limits_{x\to 3}\frac{x(x^2-x-6)}{x-3}$

$= \lim\limits_{x\to 3}\frac{x\cancel{(x-3)}(x+2)}{\cancel{x-3}}$

$= \lim\limits_{x\to 3} x(x+2)$

$= 3(3+2)$

$= 15$

$$\lim_{h\to 0}\frac{\sqrt{3+h}-\sqrt{3}}{h}\cdot\frac{\sqrt{3+h}+\sqrt{3}}{\sqrt{3+h}+\sqrt{3}}$$

$$=\lim_{h\to 0}\frac{3+h-3}{h\left(\sqrt{3+h}+\sqrt{3}\right)}$$

$$=\lim_{h\to 0}\frac{1}{\sqrt{3+h}+\sqrt{3}}$$

$$=\frac{1}{\sqrt{3}+\sqrt{3}}$$

$$=\frac{1}{2\sqrt{3}}$$

$$\lim_{x\to 0}\frac{\sqrt{1+4x}-\sqrt{1-x}}{x}\cdot\frac{\sqrt{1+4x}+\sqrt{1-x}}{}$$

$$=\lim_{x\to 0}\frac{(1+4x)-(1-x)}{x\left(\sqrt{1+4x}+\sqrt{1-x}\right)}\quad\sqrt{1+4x}+\sqrt{1-x}$$

$$=\lim_{x\to 0}\frac{5\cancel{x}}{\cancel{x}\left(\sqrt{1+4x}+\sqrt{1-x}\right)}=\frac{5}{\sqrt{1}+\sqrt{1}}=5/2$$

$$\lim_{x\to 0}\frac{\frac{4}{2+x}-2}{x}$$

$$=\lim_{x\to 0}\frac{\frac{4}{2+x}-\frac{2(2+x)}{2+x}}{x}$$

$$=\lim_{x\to 0}\frac{\frac{4-4-2x}{2+x}}{x}$$

$$=\lim_{x\to 0}\frac{-2x}{x(2+x)}$$

$$=\lim_{x\to 0}\frac{-2}{2+x}$$

$$=-1$$

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