Unit #2 Test: Vector Operations MCV4U

Name: _____

Marking Summary:

Knowledge/Understanding: questions #1, 3, 4, 5

Application: questions #2, 6, 7

Thinking/Inquiry and Problem Solving: #8, 9

Communication: all

Total Marks: _____

52

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1. Let $\vec{a} = (1,-1,1)$, $\vec{b} = (2,-3,1)$ and $\vec{c} = (-3,0,5)$. **[15 marks]**

a) Find $\vec{a} \cdot \vec{b}$

b) Find a vector perpendicular to both $\vec{a}$ and $\vec{b}$.

c) Are vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ co-planar? Justify.

d) Find the angle between vectors $\vec{a}$ and $\vec{b}$. (nearest degree)

e) Find the projection of $\vec{b}$ onto $\vec{c}$.

1 -1 1 1 -1
2 -3 1 2 -3

a) $\vec{a} \cdot \vec{b} = (1)(2) - 1(-3) + 1(1)$
$= 2 + 3 + 1$ ✓
$= 6$

b) $\vec{a} \times \vec{b} = (2, 1, -1)$ ✓✓

c) $(\vec{a} \times \vec{b}) \cdot \vec{c}$
$= (2,1,-1) \cdot (-3,0,5)$ ✓
$= -6 + 0 - 5$
$= -11$ ∴ no, not coplanar. ✓

d) $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$
$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$ ✓✓

$\cos\theta = \dfrac{6}{\sqrt{3}\sqrt{14}}$

$\theta \doteq 22.2°$

e) Proj of $\vec{b}$ onto $\vec{c}$ ✓✓
$= \dfrac{(\vec{b} \cdot \vec{c})\vec{c}}{|\vec{c}|^2} = \dfrac{(2(-3) - 0 + 5)}{(\sqrt{34})^2}(-3,0,5)$

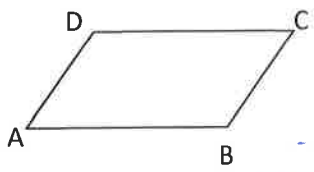
$= \dfrac{-(-3,0,5)}{34} = \left(\dfrac{3}{34}, 0, \dfrac{-5}{34}\right)$

2. *ABCD* is a parallelogram with A(-1, 3, 5), B(2, 3, 4), C(3, -1, -1) and D(0, -1, 0)

[8 marks]

a) Find the area of *ABCD*.

b) What angle do the diagonals intersect at (nearest degree)?



$\vec{AB} = (3, 0, -1)$ $\vec{AD} = (1, -4, -5)$

a) $\text{Area} = |\vec{AB} \times \vec{AD}|$

$= |(-4, 14, -12)|$

$= \sqrt{(-4)^2 + 14^2 + (-12)^2}$

$= \sqrt{356}$ sq. units

b) $\vec{AC} = (4, -4, -6)$ $\vec{BD} = (-2, -4, -4)$

$\vec{AC} \cdot \vec{BD} = 4(-2) - 4(-4) - 6(-4)$

$= 32$

$\cos\theta = \dfrac{32}{|\vec{AC}|\,|\vec{BD}|}$

$\cos\theta = \dfrac{32}{\sqrt{68}\sqrt{36}}$

$\theta \doteq 50°$

3. Consider **any** non-zero vectors $\vec{a}$ and $\vec{b}$. Which of the following statements are always true? **Circle the statements that are always true.** **[6 marks]**

a) $\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$

b) $(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{a}) = 0$

c) $(\vec{a} + \vec{b}) \cdot (\vec{a} \times \vec{b}) = 0$

d) $\vec{a} \times \vec{a} = |\vec{a}|^2$

e) $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$

f) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

4. Let $\vec{x}$ and $\vec{y}$ be vectors with $|\vec{x}| = 5$, $|\vec{y}| = 8$. The angle between $\vec{x}$ and $\vec{y}$ is 120°. Evaluate $(2\vec{x} - \vec{y}) \cdot (3\vec{x} + \vec{y})$. **[4 marks]**

$= 6|\vec{x}|^2 + 2\vec{x}\cdot\vec{y} - 3\vec{x}\cdot\vec{y} - |\vec{y}|^2$

$= 6|\vec{x}|^2 - \vec{x}\cdot\vec{y} - |\vec{y}|^2$

$= 6(5)^2 - |\vec{x}||\vec{y}|\cos 120° - 8^2$

$= 150 - (5)(8)(-\frac{1}{2}) - 64$

$= 106$

5. Let $\vec{x} = (3,4,-3)$ and $\vec{y} = (k^2, 0, 4)$. If $\vec{x}$ and $\vec{y}$ are perpendicular, then solve for k. **[4 marks]**

$\vec{x}\cdot\vec{y} = 0$

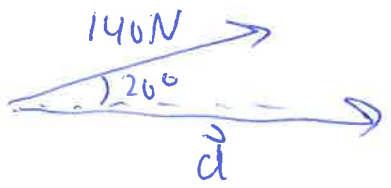
$3k^2 - 12 = 0$

$3k^2 = 12$

$k^2 = 4$

$k = \pm 2$.

6. A wagon is pulled along a road for a distance of 100 m by a 140N force applied at an angle of 20° to the road. Find the work done. **[3 marks]**



$$W = \vec{f} \cdot \vec{d}$$
$$W = |\vec{f}|\,|\vec{d}|\cos\theta^\circ$$
$$= (140N)(100m)\cos 20^\circ$$
$$\doteq 13156 N\cdot m$$
$$\doteq 13156 J$$

7. A 30N force is applied at the end of a 14 cm long wrench attached to a bolt. The force is applied at angle of 45° with the wrench. Find the magnitude of the torque acting on the bolt. **[3 marks]**

$$\vec{T} = \vec{f} \times \vec{r}$$
$$|\vec{T}| = |\vec{f}|\,|\vec{r}|\sin\theta$$
$$= 30N(0.14m)\sin 45^\circ$$
$$\doteq 3 N\cdot m \text{ or } 3J$$

8. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be vectors in 3-space. Prove that $(\vec{a}+\vec{b})\cdot\vec{c} = \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c}$. **[5 marks]**

Let $\vec{a} = (a_1, a_2, a_3)$ $\vec{b} = (b_1, b_2, b_3)$ $\vec{c} = (c_1, c_2, c_3)$

$$LS = (a_1+b_1, a_2+b_2, a_3+b_3)\cdot(c_1, c_2, c_3)$$
$$= (a_1+b_1)c_1 + (a_2+b_2)c_2 + (a_3+b_3)c_3$$
$$= a_1c_1 + b_1c_1 + a_2c_2 + b_2c_2 + a_3c_3 + b_3c_3$$
$$= a_1c_1 + a_2c_2 + a_3c_3 + b_1c_1 + b_2c_2 + b_3c_3$$
$$= \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c}$$

□.

9. Prove that $|\vec{a}\times\vec{b}| = \sqrt{(\vec{a}\cdot\vec{a})(\vec{b}\cdot\vec{b}) - (\vec{a}\cdot\vec{b})^2}$ **[4 marks]**

$$LS = |\vec{a}||\vec{b}|\sin\theta$$
$$RS = \sqrt{|\vec{a}|^2|\vec{b}|^2 - (|\vec{a}||\vec{b}|\cos\theta)^2}$$
$$= \sqrt{|\vec{a}|^2|\vec{b}|^2 - |\vec{a}|^2|\vec{b}|^2\cos^2\theta}$$
$$= \sqrt{|\vec{a}|^2|\vec{b}|^2(1-\cos^2\theta)}$$
$$= \sqrt{|\vec{a}|^2|\vec{b}|^2(\sin^2\theta)}$$
$$= |\vec{a}||\vec{b}|\sin\theta.$$

□.