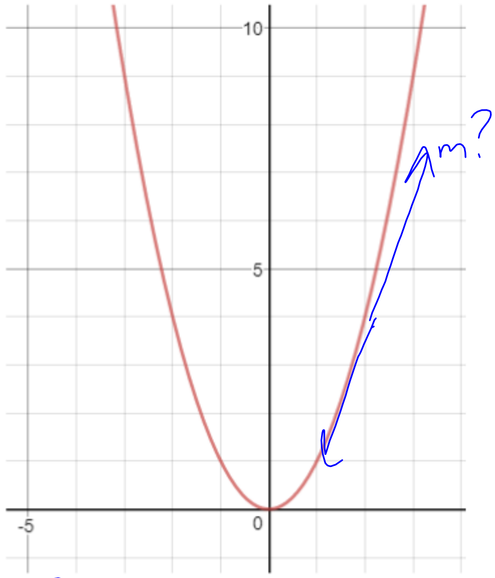
MCV4U | **The Derivative** | Lesson 2

Problem: Find the rate of change (slope) of the parabola $f(x) = x^2$ at the point (2, 4). $f(2) = 4$

$m_{tangent}$ at $(2, 4)$



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

m_{secant} from $(2,4)$ to $(2.5, f(2.5))$

$$= \frac{f(2+0.5) - f(2)}{2.5 - 2}$$

$$* = \frac{f(2+0.5) - f(2)}{0.5}$$

$$= \frac{6.25 - 4}{0.5}$$

$$= 4.5$$

try $x = 2$ to $x = 2.1$

$$* = \frac{f(2+0.1) - f(2)}{0.1}$$

$$= \frac{4.41 - 4}{0.1} = 4.1$$

$$m_{tangent} = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{(2+h)^2 - 4}{h} = \lim_{h \to 0} \frac{h^2 + 4h + 4 - 4}{h} = \lim_{h \to 0} h + 4 = 4$$

$f'(2) = 4$

We find the rate of change by a process called differentiation or by "finding the derivative".

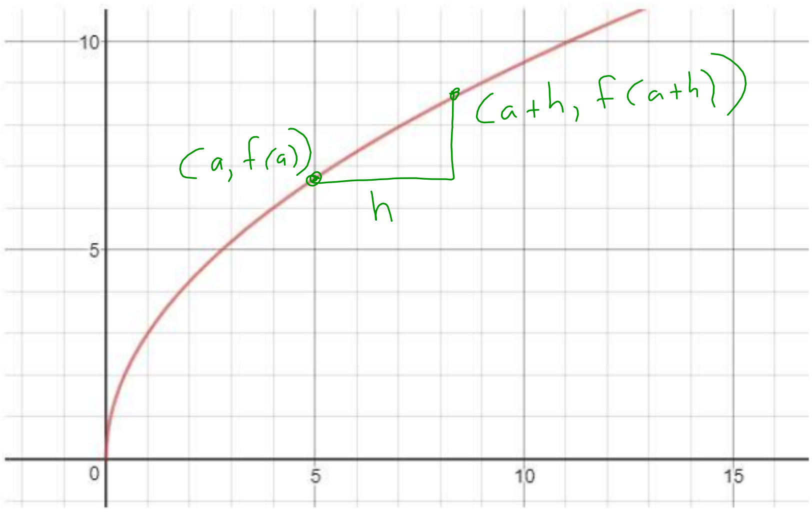
Two notations for the derivative: $\frac{dy}{dx}$ $f'(x)$

Leibniz notation

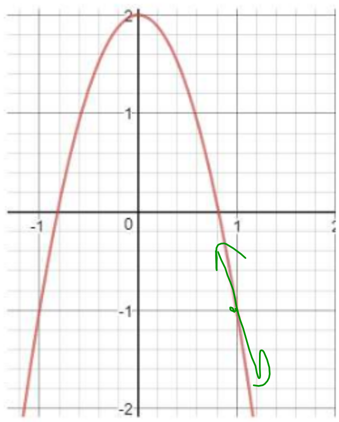
prime notation (Newton)

The *derivative of f(x) at point (a,f(a)) is given by*:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



Use the above definition to find the derivative of $f(x) = -3x^2 + 2$ at x = 1.



Find $f'(1)$

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{-3(1+h)^2 + 2 - (-1)}{h}$$

$$= \lim_{h \to 0} \frac{-3(h^2+2h+1) + 2 + 1}{h}$$

$$= \lim_{h \to 0} \frac{-3h^2 - 6h}{h}$$

$$= \lim_{h \to 0} -3h - 6$$

$$= -6$$

Also the derivative of a function *f(x)* is a new function $f'(x)$ where:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f'(x)$ or $\frac{dy}{dx}$ is called the derivative function. It allows you to calculate the derivative for any x-value. (i.e. you can substitute an 'a' value in afterwards)

Use the above definition to find the derivative function for the following.

a) $f(x) = x^3$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^2$$

$$= 3x^2$$

b) $f(x) = \sqrt{x+2}$

$$= \lim_{h \to 0} \frac{\sqrt{(x+h)+2} - \sqrt{x+2}}{h} \cdot \frac{\sqrt{x+h+2} + \sqrt{x+2}}{\sqrt{x+h+2} + \sqrt{x+2}}$$

$$= \lim_{h \to 0} \frac{(x+h+2) - (x+2)}{h\left(\sqrt{x+h+2} + \sqrt{x+2}\right)}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h+2} + \sqrt{x+2}\right)} = \frac{1}{2\sqrt{x+2}}$$

The process above is often called "first principles" (we will soon learn shortcuts)

page 49 #1cef, 2aeghi, 3f, 9, 10   (from first principles)