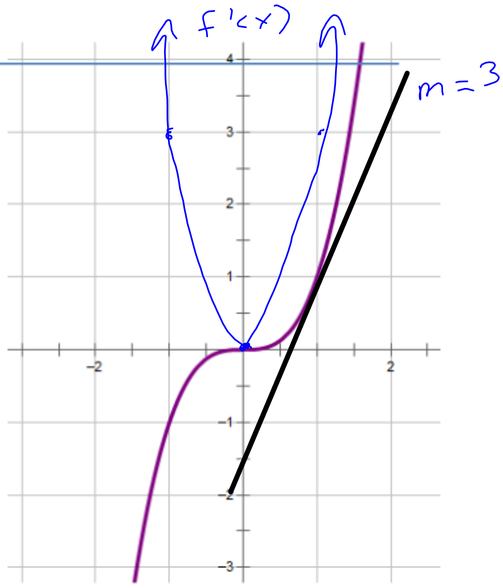
MCV4U

Derivatives & The Power Rule

Lesson 3



The Power Rule

If $y = x^n$ then $\frac{dy}{dx} = nx^{n-1}$

$f(x) = x^n,\ f'(x) = nx^{n-1}$

Example: If $f(x) = x^3$

then $f'(x) = 3x^2$

and

$f'(1) = 3(1)^2$

$= 3$

We will now prove that for $y = x^n$ then $\frac{dy}{dx} = nx^{n-1}$ for all $n \in N$. 1, 2, 3, 4...

let $f(x) = y$

$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$a = x+h \quad b = x$

$$= \lim_{h \to 0} \frac{(\cancel{x+h-x})\left((x+h)^{n-1} + (x+h)^{n-2}x + (x+h)^{n-3}x^2 + \dots + x^{n-1}\right)}{\cancel{h}}$$

$$= \lim_{h \to 0} (x+h)^{n-1} + (x+h)^{n-2}x + (x+h)^{n-3}x^2 + \dots + (x+h)x^{n-2} + x^{n-1}$$

$$= x^{n-1} + x^{n-2}x + x^{n-3}x^2 + \dots + x \cdot x^{n-2} + x^{n-1}$$

$$= \underbrace{x^{n-1} + x^{n-1} + x^{n-1} + \dots x^{n-1} + x^{n-1}}_{n \text{ terms}}$$

$$= nx^{n-1}$$

$\square$

How about $n \in I$? (negative values). Does hold. Homework question. Prove later?

Other rules for differentiation…

Constant rule: $\frac{d}{dx}(k) = 0$ where k is a constant

$f(x) = 10x^3$

Constant multiple rule: If $f(x) = kg(x)$ where k is a constant then $f'(x) = kg'(x)$.

$f'(x) = 30x^2$

Sum Rule: If $\boxed{f(x) = p(x) + q(x)}$, then $f'(x) = p'(x) + q'(x)$.

$\frac{d(ky)}{x} = k\frac{dy}{dx}$

Use the above rules to find the derivative of $f(x) = 3x^4 - 2x^2 + 9$

$f'(x) = 12x^3 - 4x$

Equations of Tangent Lines and Normals

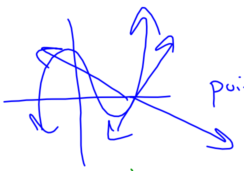
A ***tangent line*** touches a curve at only one point. The slope of a tangent line tells us the instantaneous rate of change at that point on the curve.
A ***normal*** is the line perpendicular to the tangent line at any given point on the curve.

Example: Let $f(x) = x^3 - 4x^2 + 7$. Find the equation of tangent line and the normal to the curve at point where x = 1.

$f(1) = 1^3 - 4(1)^2 + 7$

$f(1) = 4$

point $(1,4)$



$m_{tangent} = f'(1)$

$f'(x) = 3x^2 - 8x$

$f'(1) = 3 - 8$

$f'(1) = -5$

tangent $(1,4)$ $m = -5$

$-5 = \frac{y-4}{x-1} \rightarrow -5x + 5 = y - 4$

$5x + y - 9 = 0$

normal $(1,4)$ $m = 1/5$

$\frac{1}{5} = \frac{y-4}{x-1}$

$x - 1 = 5y - 20$

$x - 5y + 19 = 0$

Example Find the equation of the tangent line to $y = \frac{1}{x^2} - \frac{2}{x^3}$ at the point where x = 2.

$y = \frac{1}{x^2} - \frac{2}{x^3}$

$y = x^{-2} - 2x^{-3}$

$\frac{dy}{dx} = -2x^{-3} + 6x^{-4}$

$\frac{dy}{dx} = \frac{-2}{x^3} + \frac{6}{x^4}$

at $x = 2$

$\frac{dy}{dx} = \frac{-2}{2^3} + \frac{6}{2^4}$

$\frac{dy}{dx} = \frac{-2}{8} + \frac{6}{16}$

$\frac{dy}{dx} = 1/8$

at $x = 2$, $y = \frac{1}{2^2} - \frac{2}{2^3}$

$y = 0$

point $(2,0)$ $m = 1/8$

$\frac{1}{8} = \frac{y}{x-2}$

$8y = x - 2$

$x - 8y - 2 = 0$

Homework: text page 64 #4acehik, 6 and page 11 #7cfg, 8, **problem 2**

Also prove that if $f(x) = x^{-2}$ then $f'(x) = \frac{-2}{x^3}$ using first principles (from definition of derivative)