MCV4U

Power Rule for Rational Exponents & More Equations of Tangents

Unit 3, Lesson 4

Example 1:

Let $f(x) = \sqrt{x}$. Find $f'(x)$ from first principles.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

$f(x) = \sqrt{x}$ or $f(x) = x^{\frac{1}{2}}$

$f'(x) = \frac{1}{2\sqrt{x}}$ $\qquad$ $f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$

This result suggests that the power rule holds for rational exponents (not just integers). It does in fact hold. (You are not ready to see proof of this yet). In fact the power rule for differentiation holds for all real numbers.

Example: Find the derivative of the function $y = 3\sqrt{x} - 8\sqrt[3]{x} + \frac{4}{\sqrt[4]{x^3}}$. (Hint: convert to exponent form first)

$$y = 3x^{\frac{1}{2}} - 8x^{\frac{1}{3}} + 4x^{\frac{-3}{4}}$$

$$\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} - \frac{8}{3}x^{\frac{-2}{3}} - 3x^{\frac{-7}{4}}$$

$$= \frac{3}{2\sqrt{x}} - \frac{8}{3\sqrt[3]{x^2}} - \frac{3}{\sqrt[4]{x^7}}$$

Finding Points on a Function Given a Desired Slope

Example Find the point(s) on the curve $y = 2x^3 - 6x^2 - 45x + 1$ where the tangent line has a slope of 3.

$$\frac{dy}{dx} = 6x^2 - 12x - 45$$

when $\frac{dy}{dx} = 3$?

$$3 = 6x^2 - 12x - 45$$

$$6x^2 - 12x - 48 = 0$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$x = 4 \quad x = -2$$

$$y = x^3 + 12x^2 + 45x + 4$$

$$\frac{dy}{dx} = 3x^2 + 24x + 45$$

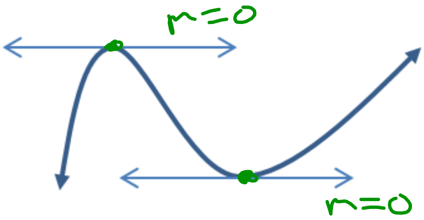
$$0 = 3x^2 + 24x + 45$$

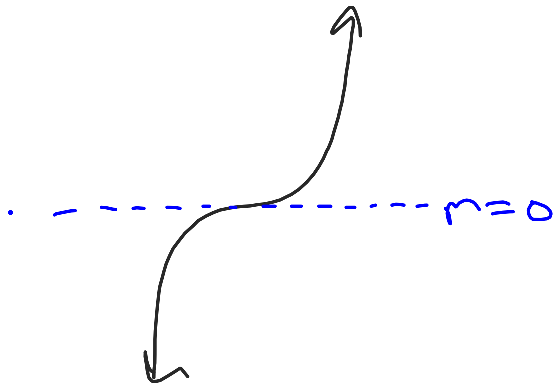
$$0 = x^2 + 8x + 15$$

$$0 = (x+5)(x+3)$$

$x = -5$ and $x = -3$

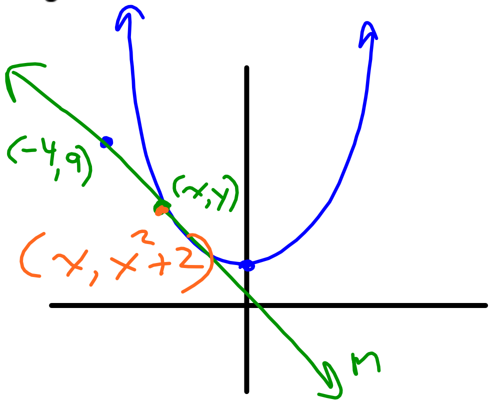
Since the tangent line has a slope of zero there is a possibility that these are turning points. As it turns out both these points are turning points.





Example: Tangent lines through points not on the curve.

A line is drawn from the point (-4, 9) so that it is a tangent to the curve $y = x^2 + 2$. At what point does this tangent line intersect the curve?



$$\frac{dy}{dx} = 2x$$

$$m_{tangent} = \frac{y-9}{x+4}$$

$$2x = \frac{y-9}{x+4}$$

but $y = x^2+2$

$$2x = \frac{(x^2+2)-9}{x+4}$$

$$2x = \frac{x^2-7}{x+4}$$

$$2x^2+8x = x^2-7$$

$$x^2+8x+7=0$$

$$(x+7)(x+1)=0$$

$$x=-7 \quad x=-1$$

$x=-1$

$y=(-1)^2+2$

$y=3$

$x=-7$

$y=(-7)^2+2$

$y=51$

$(-1, 3)$ and $(-7, 51)$

Homework: text page 64 #8, 9, 11 – 15 and page 86 #2a-d, 5a