MCV4U **The Product Rule and Power of a Function Rules** Lesson 5

In this lesson we will look at two more rules for differentiation. The first rule is called the product rule.

Example: Let $y = (x^2 + 3x)(x^2 - 7)$. We might "guess" at how to find the derivative of this function.

$$\frac{dy}{dx} \neq (2x+3)(2x)$$

$$\frac{dy}{dx} = (2x+3)(x^2-7) + (x^2+3x)(2x)$$

Is this correct?

no

We could have found the derivative of the function above using the **product rule.**

Product Rule If $f(x) = p(x)q(x)$ then OR $\dfrac{d(uv)}{dx} = \dfrac{du}{dx}v + u\dfrac{dv}{dx}$ $f'(x) = p'(x)q(x) + p(x)q'(x)$

The rule above tells us that to take the derivative of a product we take the derivative of the first factor times the second factor plus the derivative of the second factor times the first.

Proof of Product Rule

Let $f(x) = p(x)q(x)$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0}\frac{p(x+h)q(x+h) - p(x)q(x)}{h}$$

$$= \lim_{h\to 0}\frac{p(x+h)q(x+h) - p(x)q(x+h) + p(x)q(x+h) - p(x)q(x)}{h}$$

$$= \lim_{h\to 0}\frac{q(x+h)[p(x+h)-p(x)] + p(x)[q(x+h)-q(x)]}{h}$$

$$= \lim_{h\to 0} q(x+h)\lim_{h\to 0}\frac{p(x+h)-p(x)}{h} + \lim_{h\to 0} p(x)\lim_{h\to 0}\frac{q(x+h)-q(x)}{h}$$

$$= q(x)\,p'(x) + p(x)\,q'(x)$$

$$= p'(x)q(x) + p(x)q'(x)$$

$\square$

Example

Find the equation of the tangent line to $y = (x^2 + 3x - 4)(x^3 - 2x^2 - x + 7)$, where x = 1.

$$\frac{dy}{dx} = (2x+3)(x^3-2x^2-x+7) + (x^2+3x-4)(3x^2-4x-1)$$

$$\left.\frac{dy}{dx}\right|_{x=1} = 5(5) + (0)$$

at $x = 1$

$= 25$

$y = 0$

equation $25 = \frac{y}{x-1}$ $y = 25x - 25$

Expand, then use the product rule to differentiate the following. Look for a pattern.

$f(x) = (3x + 1)^2$

$f(x) = (3x+1)(3x+1)$

$f'(x) = 3(3x+1) + 3(3x+1)$

$f'(x) = 2(3x+1)(3)$

$f(x) = (5x^2 + 1)^3$

$f(x) = (5x^2+1)(5x^2+1)(5x^2+1)$

$f'(x) = 10x(5x^2+1)(5x^2+1) + (5x^2+1)(10x)(5x^2+1) + (5x^2+1)(5x^2+1)10x$

$f'(x) = 3(5x^2+1)^2 10x$

The pattern above leads to the **power of a function rule.**

Power of a Function Rule really chain rule

If $f(x) = [g(x)]^n$ then $f'(x) = n[g(x)]^{n-1}g'(x)$

OR

$$\frac{d(u^n)}{dx} = nu^{n-1}\frac{du}{dx}$$

(Note that the power rule covered before is a special case of the rule above).

Example 1: Let $y = (x^3 + 2x^2 - 1)^4$

$\frac{dy}{dx} = 4(x^3+2x^2-1)^3(3x^2+4x)$

Example 2: $y = \sqrt{(2x - 1)^3}$

$y = (2x-1)^{\frac{3}{2}}$

$\frac{dy}{dx} = \frac{3}{2}(2x-1)^{\frac{1}{2}}(2)$

$\frac{dy}{dx} = 3\sqrt{2x-1}$

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