MCV4U **The Product Rule and Power of a Function Rules** Unit 3, Lesson 5

In this lesson we will look at two more rules for differentiation. The first rule is called the product rule.

Example: Let $y = (x^2 + 3x)(x^2 - 7)$. We might "guess" at how to find the derivative of this function.

$\frac{dy}{dx} =$

Is this correct?

We could have found the derivative of the function above using the **product rule.**

Product Rule

If $f(x) = p(x)q(x)$ then OR $\frac{d(uv)}{dx} =$

$f'(x) = p'(x)q(x) + p(x)q'(x)$

The rule above tells us that to take the derivative of a product we take the derivative of the first factor times the second factor plus the derivative of the second factor times the first.

Proof of Product Rule

Let $f(x) = p(x)q(x)$ (see ***properties of limits*** on page 30 of your textbook)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example

Find the equation of the tangent line to $y = (x^2 + 3x - 4)(x^3 - 2x^2 - x + 7)$, where x = 1.

Expand, then use the product rule to differentiate the following. Look for a pattern.

$f(x) = (3x + 1)^2$

$f(x) = (5x^2 + 1)^3$

The pattern above leads to the **power of a function rule.**

Power of a Function Rule

If $f(x) = [g(x)]^n$ then $f'(x) = n[g(x)]^{n-1}g'(x)$

OR

$$\frac{d(u^n)}{dx} = nu^{n-1}\frac{du}{dx}$$

(Note that the power rule covered before is a special case of the rule above).

Example 1: Let $y = (x^3 + 2x^2 - 1)^4$

$\frac{dy}{dx} =$

Example 2: $y = \sqrt{(x-1)^3}$

$\frac{dy}{dx} =$

Text page 70 #1g, 2e, 3, 5egi, 7af, 11