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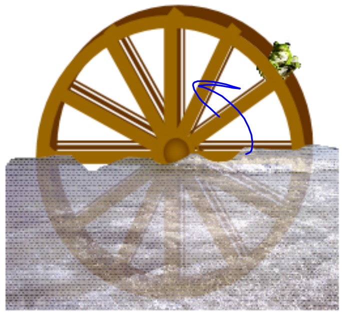
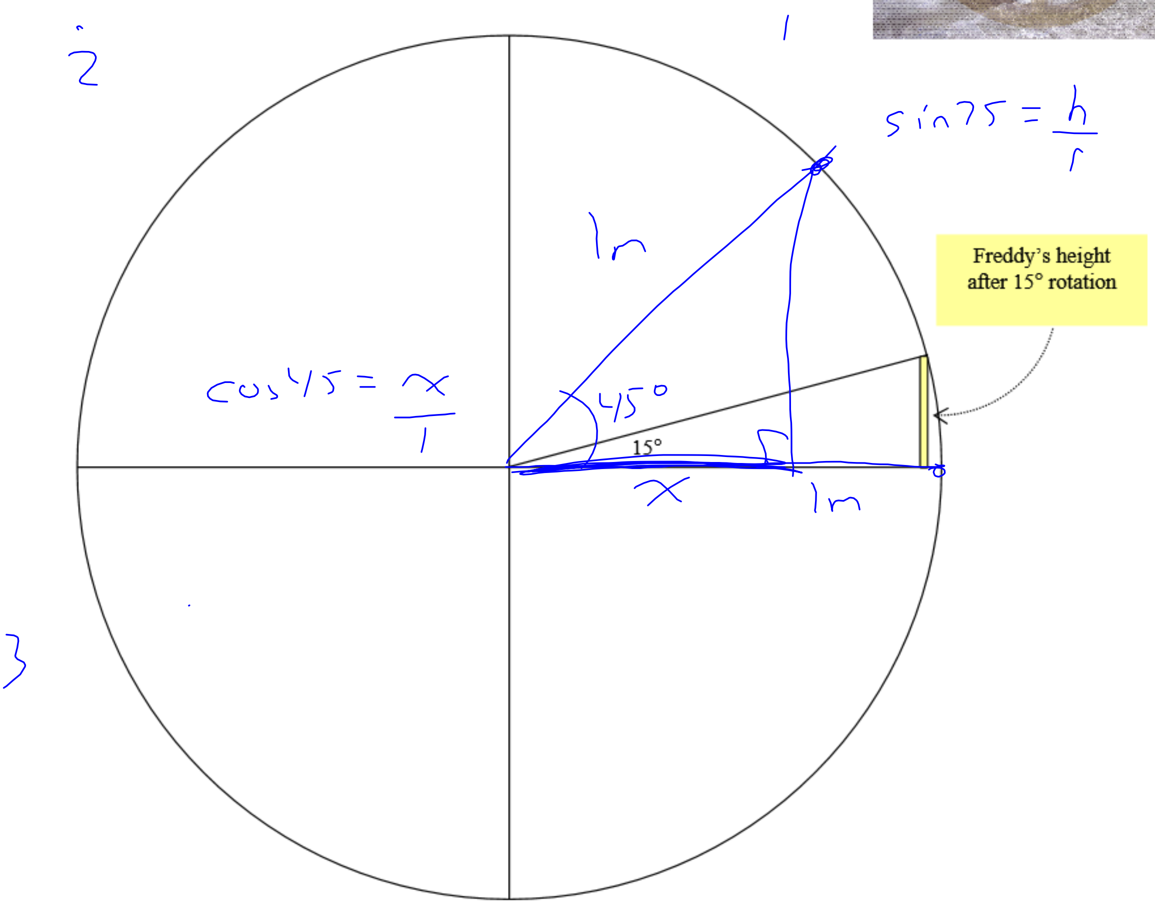
Exploring Sine & Cosine: Freddy the Frog Riding a Mill Wheel



Freddy the frog is riding on the circumference of a mill wheel on a mini-putt course as it rotates counter-clockwise. He would like to know the relationship between the angle of rotation and his height above/below the surface of the water.

A scale diagram of the wheel is shown below. The actual wheel has a radius of 1 metre.



1. Use the picture above to calculate Freddy the Frog's height at the following angles:

a) 15°
0.26m

b) 75°
0.96m

c) 90°
1m

d) 100°
$\sin 80°$
$= 0.98m$

e) 140°
0.64

f) 200°
-0.34

MCF3M

g) 270°

−1

h) 290°

−0.94

i) 300°

−0.87

j) 180°

0

2. Did you find a shortcut to calculate Freddy the Frog's height given the angle?

sin θ

same as sine!

3. For what angles is Freddy the Frog's height positive? When is it negative?

0 − 180°

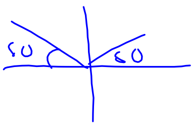
180 − 360°

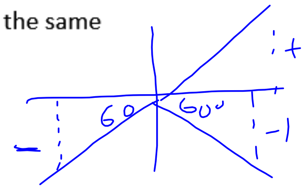
4. In 1a) you calculated Freddy's height at 15°. What other angle(s) will produce the same height?

180 − 15 = 165°

5. In 1f) you calculated Freddy's height at 100°. What other angle(s) will produce the same height?

80°





6. Complete the following:

a) sin 30° = sin 150°

b) sin 135° = sin 45

c) sin 240° = −sin 60

= sin 300°

7. Suppose we want to calculate the Freddy's horizontal distance from the centre of the wheel. We will say his distance is positive if he is to the right of the wheel and negative if he is to the left of the wheel.

Calculate his horizontal distance for the following angles.

a) 0°

1m

b) 45°

0.71m

c) 150°

−0.87

d) 200°

−0.93

e) 270°

0

f) 330°

0.87m

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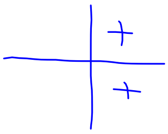
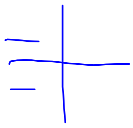
8. Did you find a shortcut to calculate Freddy's horizontal distance based on the angle of rotation?

$\cos\theta$

9. For what angles is Freddy the Frog's horizontal distance positive? When is it negative?

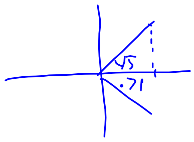


0 − 90
270 − 360



90° − 270°

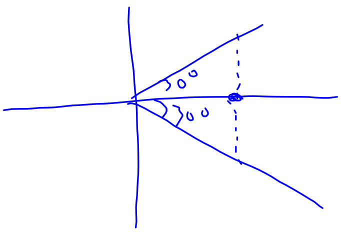
10. In 7b) you calculated Freddy's horizontal distance at an angle of 45°. For what angle will he have the same horizontal distance?

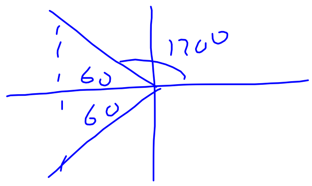


360 − 45
= 315°

11. Complete the following:

a) cos 30° = cos 330° b) cos 350° = cos 10° c) cos 120° = cos 240°





Can you determine whether the following ratios are positive or negative? (without a calculator)

sin 30°
+

sin 150°
+

sin 210°
−

cos 110°
−

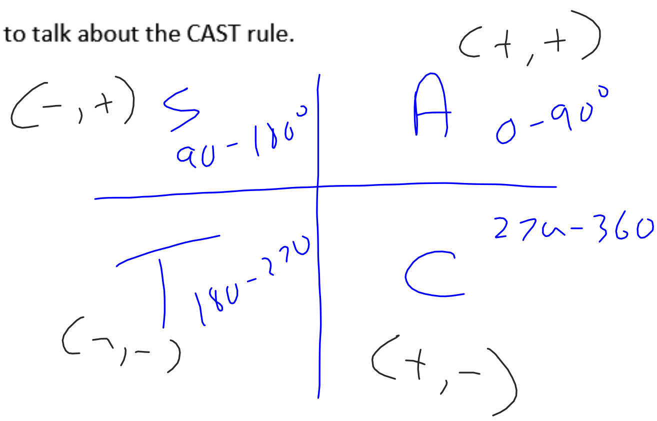
cos 300°
+

tan 120°
− = −tan 60°

tan 250°
+

tan 45°
+

We can use the patterns above to talk about the CAST rule.



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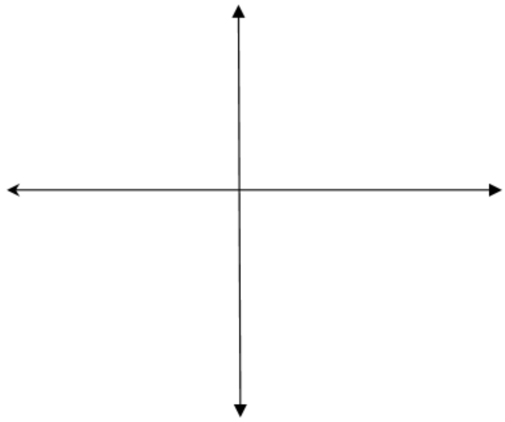


So what is really going on here?

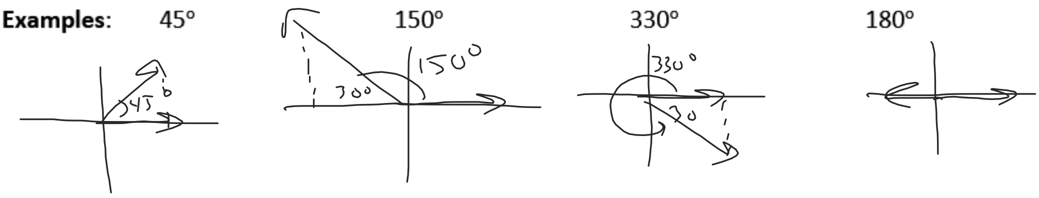
The Formal Definition

To define trigonometric ratios for all angles we first draw the angle in standard position.

An angle in standard position has it vertex at the origin and one arm (initial arm) on the positive x-axis. The other arm is called the terminal arm.

Angles are measured with a counter-clockwise rotation being positive.

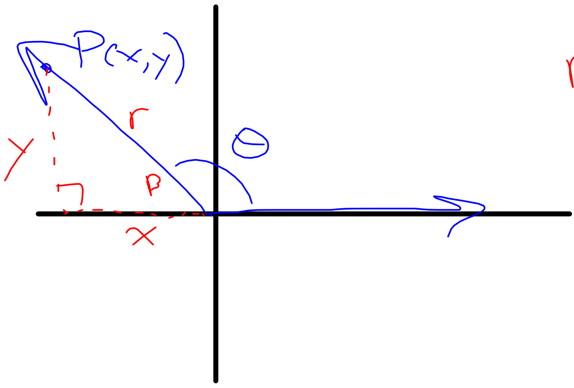
Examples: 45° 150° 330° 180°



We call β the related acute angle. It is the angle formed with the nearest x-axis.

Let P(x,y) be any point on the terminal arm. Let r be the distance from P to the origin (always positive).

Then:



$r^2 = x^2 + y^2$

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

$r > 0$

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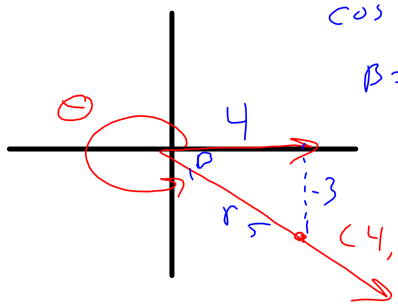
Example Suppose that (4,-3) is a point on a terminal arm of an angle θ in standard position. Find the sine, cosine and tangent of θ. Solve for θ.

$\sin\theta = -\frac{3}{5}$

$\cos\theta = 4/5$

$\tan\theta = -3/4$



$\cos\beta = \frac{4}{5}$

$\beta = \cos^{-1}(4/5)$

$\beta \doteq 37°$

$r^2 = (-3)^2 + 4^2$

$r^2 = 25$

$r = 5$

$\therefore \theta = 360 - 37$ or $\theta \doteq 323°$

Example Suppose that (-1,-5) is a point on a terminal arm of an angle θ in standard position. Find the sine, cosine and tangent of θ. Solve for θ.

Homework

1. Sketch the angle θ that goes through each point in standard position. Then find the sine, cosine and tangent ratios. Solve for θ. (Nearest degree)

   a) (4, -1) b) (-3, -4) c) (-2, -5)

2. Fill in the blanks.

   a) sin 60° = sin ____ b) cos 300° = cos _____ c) tan 60° = tan _____.

3. Given that cos 60° = 0.5, find the following (without a calculator).

   a) cos 300° b) cos 120° c) cos 240°

4. Text page 245 #4abc