Example – Find the derivative of the function $f(x) = (x^2+3)^3(3x-11)^4$. Leave your answer in simplified factored form. Then find any ***critical points***.

Critical point – a point on the graph where the derivative does not exist, or is equal to zero.

$$f'(x) = 3(x^2+3)^2(2x)(3x-11)^4 + (x^2+3)^3(4)(3x-11)^3(3)$$

$$f'(x) = 6x(x^2+3)^2(3x-11)^4 + 12(x^2+3)^3(3x-11)^3$$

$$f'(x) = 6(x^2+3)^2(3x-11)^3\left[x(3x-11) + 2(x^2+3)\right]$$

$$= 6(x^2+3)^2(3x-11)^3(5x^2-11x+6)$$

$$f'(x) = 6(x^2+3)^2(3x-11)^3(5x-6)(x-1)$$

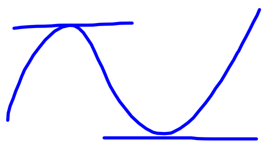
$f'(x)=0$

$$0 = 6(x^2+3)^2(3x-11)^3(5x-6)(x-1)$$

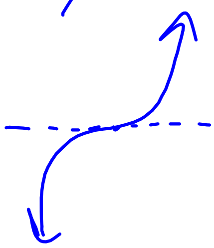
$x^2+3 \neq 0$ $\quad 3x-11=0$, $x = 11/3$ $\quad 5x-6=0$, $x = 6/5$ $\quad x=1$

critical points at $x = 11/3, 6/5, 1$

Are all critical points equal to turning points?



$y = x^3 \quad y' = 3x^2$



Example 2 – Find the slope of the tangent line where x = 1 for $f(x) = \frac{1}{\sqrt{4x^2+5x}}$

find $f'(1)$

$f(x) = (4x^2+5x)^{-1/2}$

$f'(x) = -\frac{1}{2}(4x^2+5x)^{-3/2}(8x+5)$

$f'(x) = \frac{-(8x+5)}{2(4x^2+5x)^{3/2}}$

$f'(x) = \frac{-8x-5}{2(\sqrt{4x^2+5x})^3}$

$f'(1) = \frac{-8-5}{2(\sqrt{9})^3}$

$= \frac{-13}{54}$

Note that if $f(x) = \frac{p(x)}{q(x)}$ then the product and power rules can be used to find $f'(x)$. HOW?

$f(x) = p(x)[q(x)]^{-1}$

Example Find *f′(x)* for $f(x) = \frac{2x-5}{x^2-3x}$.

$f(x) = (2x-5)(x^2-3x)^{-1}$

$f'(x) = 2(x^2-3x)^{-1} + (2x-5)(-1)(x^2-3x)^{-2}(2x-3)$

$f'(x) = (x^2-3x)^{-2}[2(x^2-3x) - (2x-5)(2x-3)]$

$f'(x) = (x^2-3x)^{-2}(-2x^2+10x-15)$

$f'(x) = \frac{-2x^2+10x-15}{(x^2-3x)^2}$