MCV4U **The Quotient Rule** Lesson 7

The Quotient Rule

If $f(x) = \frac{p(x)}{q(x)}$ then $f'(x) = \frac{p'(x)q(x) - p(x)q'(x)}{[q(x)]^2}$

OR $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx}v - u\frac{dv}{dx}}{v^2}$

Consider the example from last day: $f(x) = \frac{2x-5}{x^2-3x}$. We can use the quotient rule instead of the power rule.

$$f'(x) = \frac{2(x^2-3x) - (2x-5)(2x-3)}{(x^2-3x)^2}$$

$$f'(x) = \frac{-2x^2+10x-15}{(x^2-3x)^2}$$

Proof of the Quotient Rule

$f(x) = \frac{p(x)}{q(x)}$

$$f(x)q(x) = p(x)$$

$$p(x) = f(x)q(x)$$

$$p'(x) = f'(x)q(x) + f(x)q'(x)$$

$$f'(x) = \frac{p'(x) - f(x)q'(x)}{q(x)}$$

$$f'(x) = \frac{p'(x) - \frac{p(x)}{q(x)}q'(x)}{q(x)} \times \frac{q(x)}{q(x)}$$

$$f'(x) = \frac{p'(x)q(x) - p(x)q'(x)}{[q(x)]^2}$$

Example: Differentiate $y = \frac{2x-3}{4x+1}$

$$\frac{dy}{dx} = \frac{2(4x+1)-(2x-3)4}{(4x+1)^2}$$

$$= \frac{14}{(4x+1)^2}$$

Sometimes the quotient rule is not always the best option for dealing with rational expressions.

Example $y = \frac{-6}{(x+3)^4}$

$$y = -6(x+3)^{-4}$$

$$\frac{dy}{dx} = +24(x+3)^{-5}$$

$$= \frac{+24}{(x+3)^5}$$

$$\frac{dy}{dx} = \frac{0(x+3)^4 - (-6)(4)(x+3)^3}{(x+3)^8}$$

Example $f(x) = \frac{(5x-3)^3}{(x+2)^5}$

$$f'(x) = \frac{3(5x-3)^2(5)(x+2)^{\cancel{5}\,4} - (5x-3)^3(5)\cancel{(x+2)^4}}{(x+2)^{\cancel{10}\,6}}$$

$$f'(x) = \frac{15(5x-3)^2(x+2) - 5(5x-3)^3}{(x+2)^6}$$

$$f'(x) = \frac{5(5x-3)^2\left(3(x+2)-(5x-3)\right)}{(x+2)^6}$$

$$f'(x) = \frac{5(5x-3)^2(-2x+9)}{(x+2)^6}$$

Example $f(x) = \sqrt[3]{\frac{7x+1}{30x-3}}$

$$f(x) = \left(\frac{7x+1}{30x-3}\right)^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3}\left(\frac{7x+1}{30x-3}\right)^{-\frac{2}{3}}\left(\frac{7(30x-3)-(7x+1)30}{(30x-3)^2}\right)$$

$$= \frac{1}{\cancel{3}}\;\frac{(30x-3)^{\frac{2}{3}}}{(7x+1)^{\frac{2}{3}}}\left(\frac{-\cancel{51}\,17}{(30x-3)^2}\right)$$

$$= \frac{-17(30x-3)^{-\frac{4}{3}}}{(7x+1)^{\frac{2}{3}}}$$

$$= \frac{-17}{\sqrt[3]{(7x+1)^2(30x-3)^4}}$$

Text page 79 #2g, 3d, 4ab, 5h, 6g, 9d, 11c and page 86 #2n, 3dg, 5c