# Assignment 2 - Solutions

1. a) $$f'(x) = \lim_{h \to 0} \frac{\frac{2(x+h)+1}{x+h-1} - \frac{2x+1}{x-1}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(2x+2h+1)(x-1) - (x+h-1)(2x+1)}{h(x+h-1)(x-1)}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^2} + \cancel{2xh} + \cancel{x} - \cancel{2x} - 2h - \cancel{1} - (\cancel{2x^2} + \cancel{x} + \cancel{2xh} + h - \cancel{2x} - \cancel{1})}{h(x+h-1)(x-1)}$$

$$= \lim_{h \to 0} \frac{-3\cancel{h}}{\cancel{h}(x+h-1)(x-1)}$$

$$= \frac{-3}{(x-1)(x-1)}$$

$$= \frac{-3}{(x-1)^2}$$

b) $f(x) = (2x+1)(x-1)^{-1}$

$f'(x) = 2(x-1)^{-1} + (2x+1)(-1)(x-1)^{-2}$

$f'(x) = \frac{2}{x-1} - \frac{(2x+1)}{(x-1)^2}$

$f'(x) = \frac{2(x-1) - (2x+1)}{(x-1)^2}$

$f'(x) = \frac{-3}{(x-1)^2}$

c) $f'(x) = \frac{2(x-1) - (2x+1)}{(x-1)^2}$

$f'(x) = \frac{-3}{(x-1)^2}$.

d) $f'(x) \neq 0$. There are no critical points

e) $f(4) = \frac{2(4)+1}{4-1} = 3$ $(4,3)$

$f'(4) = \frac{-3}{(4-1)^2} = \frac{-3}{9} = -1/3$.

$m = -1/3$ point $(4,3)$

Equation $\quad \frac{-1}{3} = \frac{y-3}{x-4}$

$-x+4 = 3y-9$

$x+3y-13=0$ or $y = -\frac{1}{3}x + \frac{13}{3}$.

2.

a) $y = (1-2x^3)(x^2-2)^2$

$\frac{dy}{dx} = -6x^2(x^2-2)^2 + (1-2x^3)(2)(x^2-2)(2x)$

$\frac{dy}{dx} = -6x^2(x^2-2)^2 + 4x(1-2x^3)(x^2-2)$

$\frac{dy}{dx} = -2x(x^2-2)(3x(x^2-2) - 2(1-2x^3))$

$\frac{dy}{dx} = -2x(x^2-2)(3x^3-6x-2+4x^3)$

$\frac{dy}{dx} = -2x(x^2-2)(7x^3-6x-2)$

b) $y = 3x\sqrt[3]{3x^2-1}$

$y = 3x(3x^2-1)^{\frac{1}{3}}$

$\frac{dy}{dx} = 3(3x^2-1)^{\frac{1}{3}} + 3x\left(\frac{1}{3}\right)(3x^2-1)^{-\frac{2}{3}}(6x)$

$\frac{dy}{dx} = 3(3x^2-1)^{\frac{1}{3}} + 6x^2(3x^2-1)^{-\frac{2}{3}}$

$\frac{dy}{dx} = 3(3x^2-1)^{-\frac{2}{3}}[3x^2-1 + 6x^2]$

$\frac{dy}{dx} = \frac{3(9x^2-1)}{(\sqrt{3x^2-1})^2}$ or $= \frac{27x^2-3}{\sqrt{(3x^2-1)^2}}$

c) $y = \dfrac{3x^2 - x}{2x^2 - x}$

$$\frac{dy}{dx} = \frac{(6x-1)(2x^2-x) - (3x^2-x)(4x-1)}{(2x^2-x)^2}$$

$$\frac{dy}{dx} = \frac{12x^3 - 6x^2 - 2x^2 + x - 12x^3 + 3x^2 + 4x^2 - x}{(2x^2-x)^2}$$

$$\frac{dy}{dx} = \frac{-x^2}{(2x^2-x)^2}$$

d) $f(x) = \left(\dfrac{2x-1}{x^3}\right)^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2}\left(\frac{2x-1}{x^3}\right)^{-\frac{1}{2}}\left(\frac{2x^3 - (2x-1)3x^2}{x^6}\right)$$

$$f'(x) = \frac{1}{2}\left(\frac{x^3}{2x-1}\right)^{\frac{1}{2}}\left(\frac{2x^3 - 6x^3 + 3x^2}{x^6}\right)$$

$$f'(x) = \frac{1}{2}\left(\frac{x^{3/2}}{\sqrt{2x-1}}\right)\left(\frac{3x^2 - 4x^3}{x^6}\right)$$

$\frac{3}{2} - 6$

$= \frac{3}{2} - \frac{12}{2}$

$= -\frac{9}{2}$

$$f'(x) = \frac{3x^2 - 4x^3}{\sqrt{x^9}\,\sqrt{2x-1}}$$

4. $V(t)=90\left(1-\frac{t}{18}\right)^2$

a) $V(0)=90$

$0=90\left(1-\frac{t}{18}\right)^2$

$0=1-\frac{t}{18}$

$\frac{t}{18}=1$

$t=18.$

$D: \{t\in R \mid 0\le t\le 18\}$

$V: \{V(t)\in R \mid 0\le V(t)\le 90\}$

b) $V'(t)=180\left(1-\frac{t}{18}\right)\left(-\frac{1}{18}\right)$

$V'(t)=-10\left(1-\frac{t}{18}\right)$

$V'(12)=-10\left(1-\frac{12}{18}\right)$

$=-10\left(\frac{6}{18}\right)$

$=\frac{-10}{3}$ L/h.

c) $40=90\left(1-\frac{t}{18}\right)^2$

$\frac{4}{9}=\left(1-\frac{t}{18}\right)^2$

$\pm\frac{2}{3}=\left(1-\frac{t}{18}\right)$

$-1\pm\frac{2}{3}=\frac{-t}{18}$

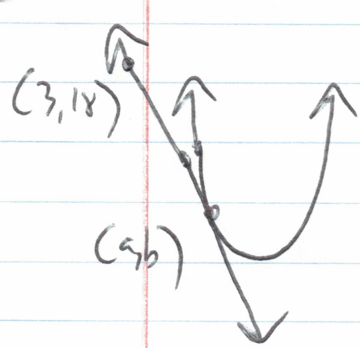
$18\pm12=t$

$t \neq 30$ $\qquad t = 6$

$t < 18$

$$V'(6) = -10\left(1 - \frac{6}{18}\right)$$
$$= -10\left(\frac{12}{18}\right)$$
$$= -10\left(\frac{2}{3}\right)$$
$$= -\frac{20}{3}\ L/\text{hour.}$$

5. a) $f(x) = -x^2 + 6x + 5$




Let $(a, b)$ be point of tangency on $f(x)$. but $b = -a^2 + 6a + 5$

$\therefore (a, -a^2 + 6a + 5)$ is point on $f(x)$

$f'(x) = -2x + 6 \rightarrow f'(a) = -2a + 6$

$m_{tangent} = -2a + 6$ and $m_{tangent} = \dfrac{b - 18}{a - 3}$

$$\therefore \quad -2a + 6 = \frac{-a^2 + 6a + 5 - 18}{a - 3}$$

$$(-2a + 6)(a - 3) = -a^2 + 6a - 13$$

$$-2a^2 + 12a - 18 = -a^2 + 6a - 13$$

$$0 = a^2 - 6a + 5$$

$$(a - 5)(a - 1) = 0$$

$$a = 5 \text{ or } a = 1$$

$a = 5$

$f(5) = 10$

$f'(5) = -4$

equation

$-4 = \dfrac{y - 10}{x - 5}$

$\boxed{y = -4x + 30}$

$a = 1$ $\quad f'(1) = 4$

$f(1) = 10$

$4 = \dfrac{y - 10}{x - 1}$

$\boxed{y = 4x + 6}$

5 b) $y = \frac{5x+2}{x+2}$

$$\frac{dy}{dx} = \frac{5(x+2) - (5x+2)(1)}{(x+2)^2}$$

$$\frac{dy}{dx} = \frac{8}{(x+2)^2}$$

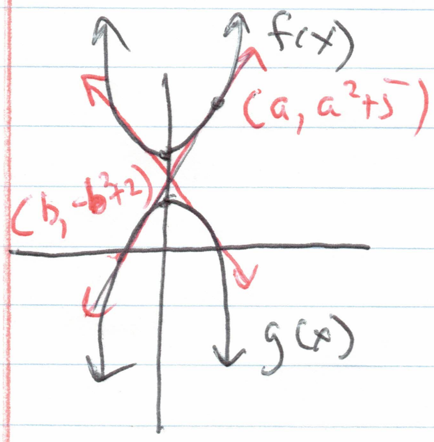
$\frac{dy}{dx} > 0$ (always)

since $8 > 0$

$(x+2)^2 > 0$

$\therefore$ $y = \frac{5x+2}{x+2}$ is always increasing.

6. $f(x) = x^2+5 \qquad g(x) = -x^2+2$




Consider a common tangent line, from point $(a, a^2+5)$ on $f(x)$ to point $(b, -b^2+2)$ on $g(x)$.

$f'(a) = g'(b)$

$2a = -2b$

$a = -b$

so our 2 points are really $(b, -b^2+2)$ and $(-b, b^2+5)$*

*since $(-b)^2 = b^2$

$m_{tangent} = \dfrac{b^2+5-(-b^2+2)}{-b-b}$

$-2b = \dfrac{b^2+5+b^2-2}{-2b}$

$4b^2 = 2b^2+3$

$2b^2 = 3$

$b = \pm\sqrt{3/2}$

$b = \pm\dfrac{\sqrt{6}}{2}$

If $b = \dfrac{\sqrt{6}}{2}$, $a = -\dfrac{\sqrt{6}}{2}$

$g'\left(\frac{\sqrt{6}}{2}\right) = -\sqrt{6}$

point is $\left(-\frac{\sqrt{6}}{2}, \frac{13}{2}\right)$

Equation is $-\sqrt{6} = \dfrac{y - 13/2}{x + \sqrt{6}/2}$

$-\sqrt{6}x - 3 = y - 13/2$

$\boxed{y = -\sqrt{6}x + 7/2}$

If $b = -\frac{\sqrt{6}}{2}$, $a = \frac{\sqrt{6}}{2}$

$f'\left(\frac{\sqrt{6}}{2}\right) = \sqrt{6}$ point is $\left(\frac{\sqrt{6}}{2}, \frac{13}{2}\right)$

Equation is: $\sqrt{6} = \dfrac{y - 13/2}{x - \sqrt{6}/2}$

$\sqrt{6}x - 3 = y - 13/2$

$\boxed{y = \sqrt{6}x + 7/2}$