MCF3M

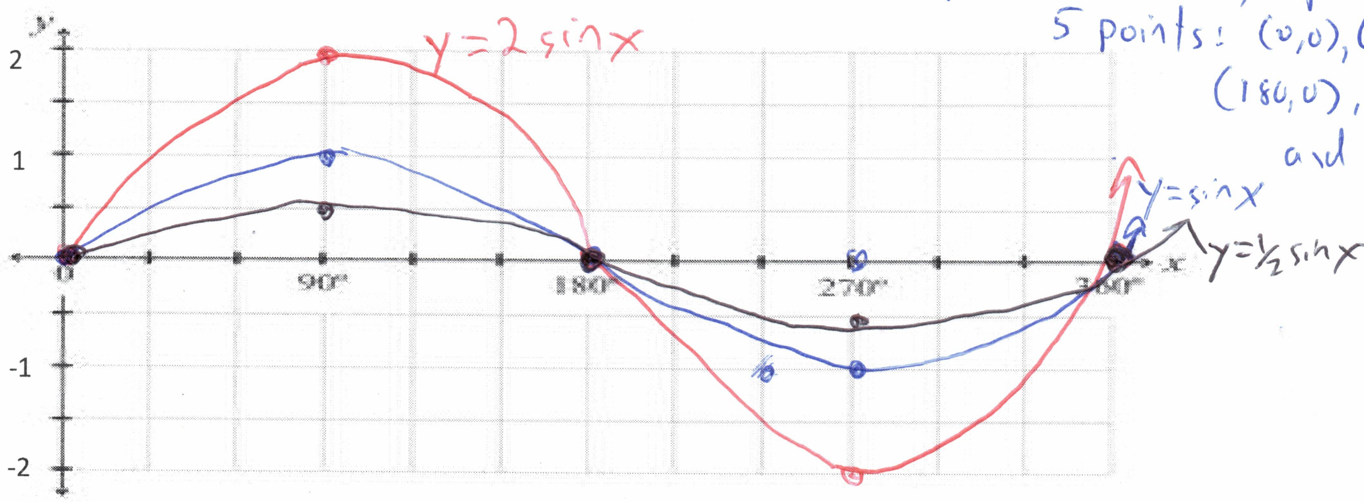
Transformations of the Sine Function

Today we will look at all sorts of graphs that have the sine ratio in their equation. We will talk about transformations of y = sin x, similar to the way we discussed transformations of y = x^2 when we discussed vertex form.

Exploring Graphs of the From y = a sin x

* notice you can sketch y = sinx just by plotting 5 points: (0,0), (90,1), (180,0), (270,-1) and (360,0)

Sketch a graph of $y = \sin x$ below. *



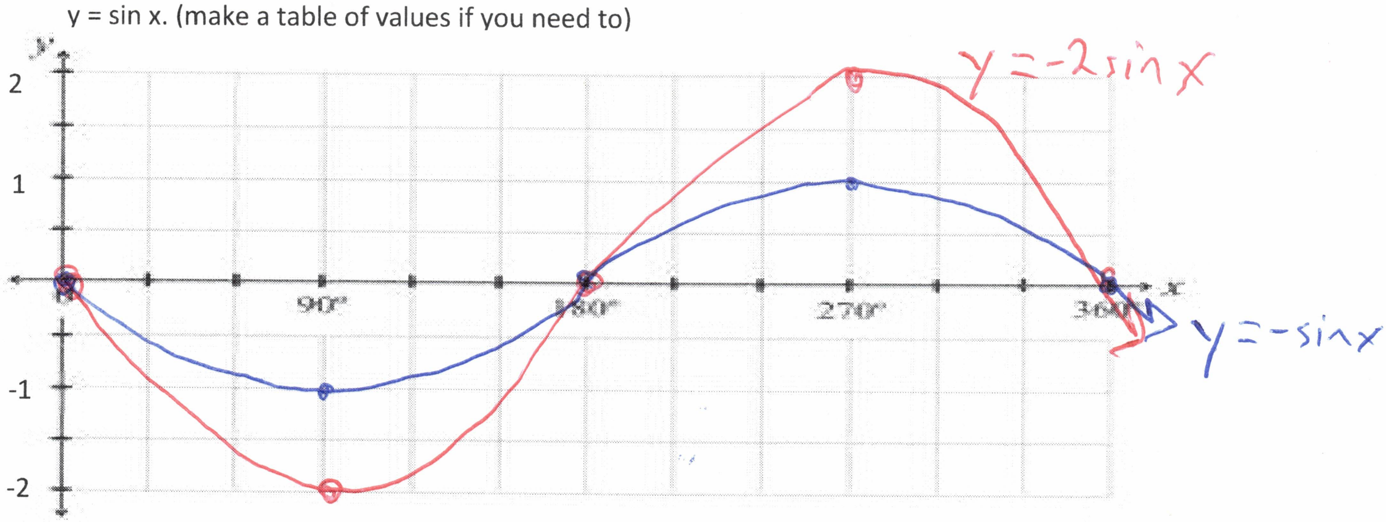
1) What might the graph of $y = 2\sin x$ look like? Sketch this graph above as well. (make a table of values if you need to)

vertical stretch factor 2

2) What might the graph of $y = 0.5\sin x$ look like? Sketch this graph above as well. (make a table of values if you need to)

vertical compression by ½.

3) Sketch the graphs of **y = - sin x** and **y = -2 sin x** on the graphs below, by considering how they would compare to y = sin x. (make a table of values if you need to)



"reflected in the x-axis"

Summarize the results from the 5 graphs above by completing this table. Can you use a pattern to complete the last 2 rows of the table?

Equation	Amplitude	Equation of Axis	Period
$y = \sin x$	1	$y=0$	$360°$
$y = 2\sin x$	2	$y=0$	$360°$
$y = 0.5\sin x$	0.5	$y=0$	$360°$
$y = -\sin x$	1	$y=0$	$360°$
$y = -2\sin x$	2	$y=0$	$360°$
$y = 10\sin x$	10	$y=0$	$360°$
$y = -12\sin x$	12	$y=0$	$360°$

axis is at $y = \frac{\max + \min}{2}$

(amplitude is $\frac{\max - \min}{2}$ (height of a wave)

What lesson is learned here about the graph of y = a sin x ?

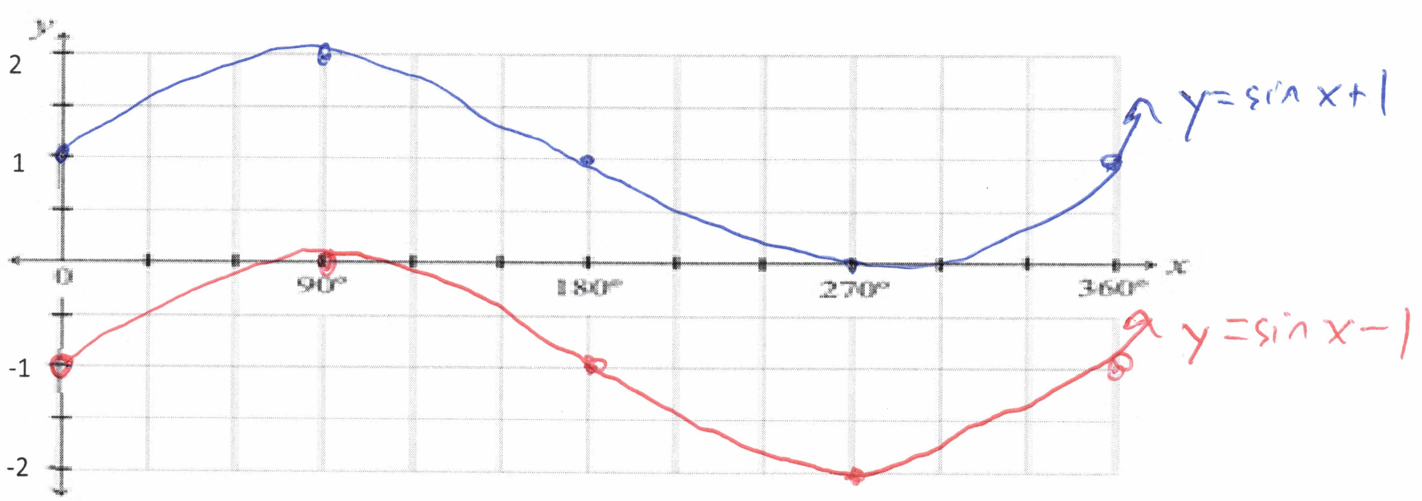
It is the graph of $y = \sin x$ stretched/compressed factor 'a'.

The amplitude of the graph is 'a'.

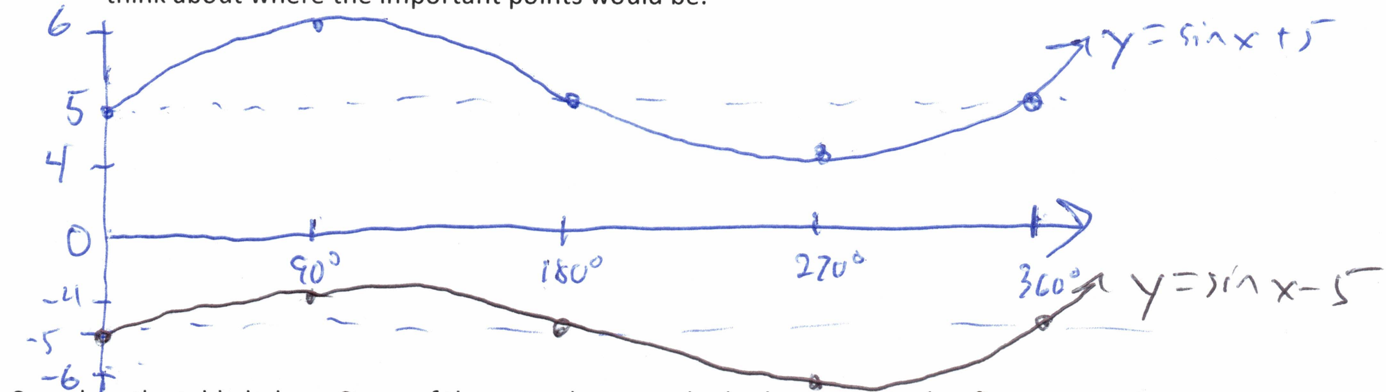
Exploring Graphs of y = sin x + c

1) What would the graph of $y = \sin x + 1$ look like compared to $y = \sin x$? Use this information to sketch $y = \sin x + 1$ below. shift up 1 unit

2) What would the graph of $y = \sin x - 1$ look like compared to $y = \sin x$? Use this information to sketch $y = \sin x - 1$ below. shift down 1 unit



3) Now sketch the graphs of y = sin x + 5 and y = sin x – 5. Notice that you don't need graph paper to do this, just think about where the important points would be.



Complete the table below. Some of these you have graphed others you need to figure out.

Equation	Amplitude	Equation of Axis	Period
$y = \sin x$	1	y=0	360°
$y = \sin x + 1$	1	y=1	360°
$y = \sin x - 1$	1	y=-1	360°
$y = \sin x + 5$	1	y=5	360°
$y = \sin x - 5$	1	y=-5	360°
$y = \sin x + 20$	1	y=20	360°
$y = 12\sin x + 6$	12	y=6	360°

What is the lesson learned about graphs of the form $y = \sin x + c$?

It is the graph of y= sin x shifted up/down by 'c'.
Axis at y=c.

Exploring Graphs of $y = \sin(kx)$

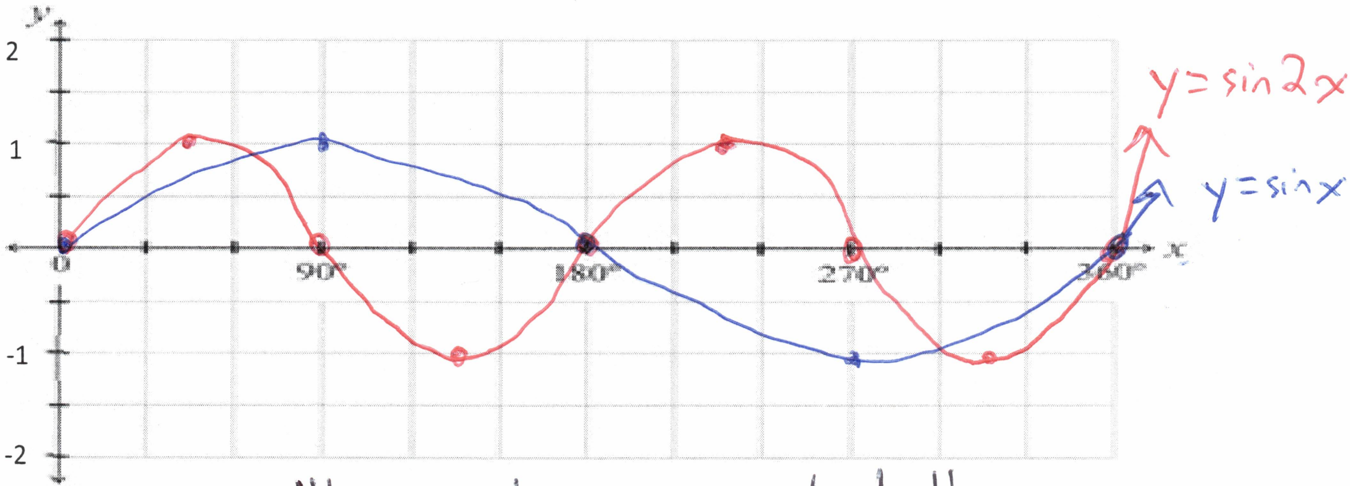
We know what the graph of y = sin x looks like, but what about the graph of y = sin 2x ? This question is a little more challenging than the ones above.

1) To answer this question, try completing a table of values.

x	y = sin x	y = sin 2x
0	0	0
45	0.71	1
90	1	0
135	0.71	-1
180	0	0

x	y = sin x	y = sin 2x
225	-0.71	1
270	-1	0
315	-0.71	-1
360	0	0

2) Sketch y = sin x and y = sin 2x below.



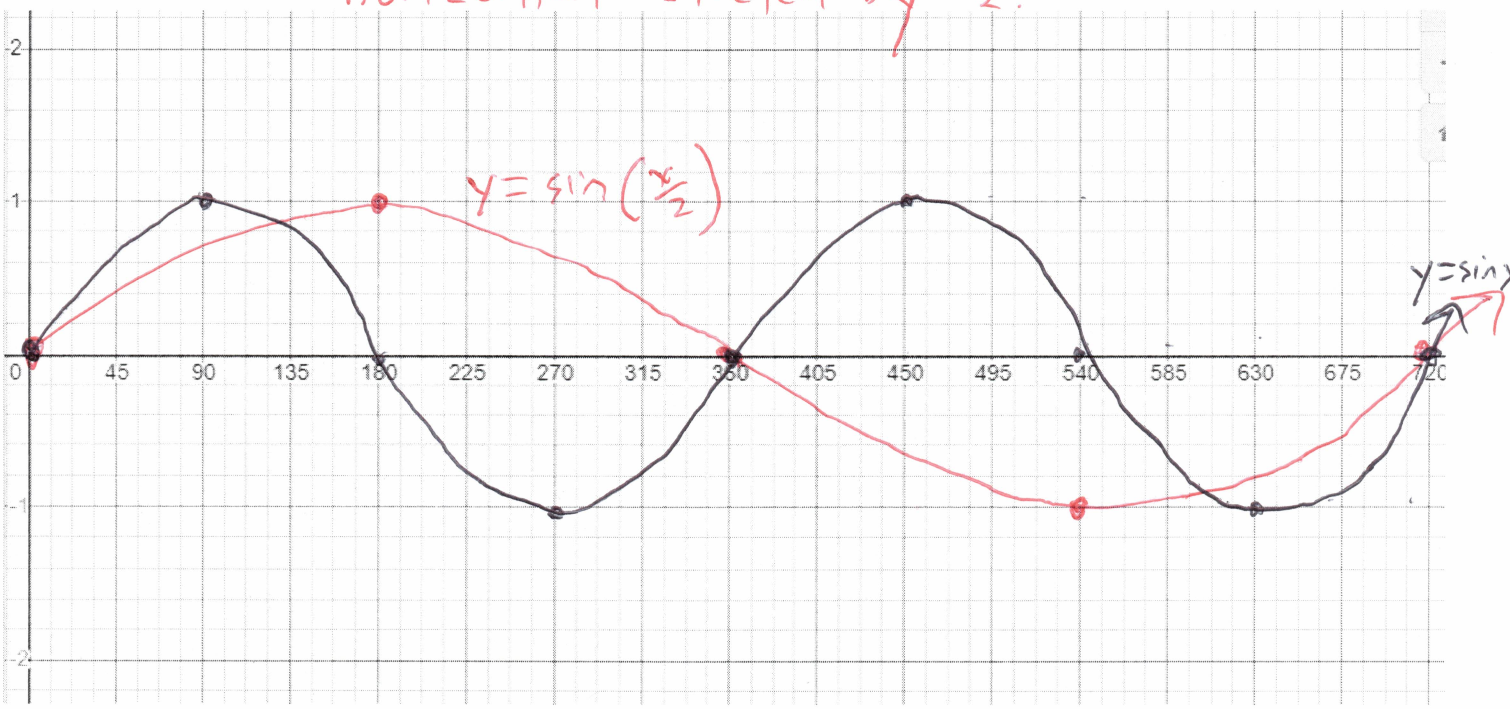
"horizontal compression by $\frac{1}{2}$"

3) Try to figure out what $y = \sin\left(\frac{x}{2}\right)$ looks like. Note that $y = sin\left(\frac{x}{2}\right)$ is the same as $y = sin\left(\frac{1}{2}x\right)$

x	y = sin x	$y = \sin\left(\frac{x}{2}\right)$
0	0	0
90	1	0.71
180	0	1
270	-1	0.71
360	0	0
450	1	-0.71
540	0	-1
630	-1	-0.71
720	0	0

Sketch $y = \sin x$ and $y = \sin\left(\frac{x}{2}\right)$ graphs below.

horizontal stretch by 2.



Complete the table below. See if you notice the pattern and can complete the table for functions you did not graph.

Equation	Amplitude	Equation of Axis	Period
$y = \sin x$	1	$y=0$	360°
$y = \sin(2x)$	1	$y=0$	180°
$y = \sin\left(\frac{x}{2}\right)$	1	$y=0$	720°
$y = \sin(3x)$	1	$y=0$	$360 \div 3 = 120$
$y = \sin\left(\frac{x}{10}\right)$	1	$y=0$	$360 \div \frac{1}{10} = 360 \times 10 = 3600$

$y = \sin(kx)$ is $y = \sin x$ with a horizontal stretch by k.

The period of $y = \sin(kx)$ is $\frac{360^\circ}{k}$.