MCF3M

# Modeling with Sinusoidal Functions

Sinusoidal functions are used to model many situations involving periodic behavior.

**Example 1**

A person begins to ride a ferris wheel and starts a stop watch. The person's height on the ferris wheel can be modeled by the equation $h(t) = -10\cos[1.5t] + 11$ where *h(t)* is their height (in m) based on the time on the stopwatch, $t$, in seconds.

a) Sketch this graph for 2 cycles.

b) How high is the person when they first get on the ferris wheel?

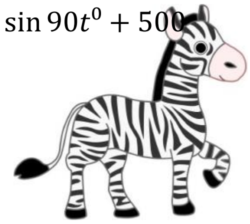
c) What do the amplitude, equation of axis and period represent in this question?

d) Another ferris wheel has a radius of 13 m and a lowest point 2 m from the ground. It takes 120 seconds to make one full revolution and a rider boards the ferris wheel at its lowest point. What would the equation of this ferris wheel be?

MCF3M

**Example 2**

Suppose that the population of zebra in a certain area is given by the function $P(t) = 250\sin 90t^0 + 500$ where *P(t)* is the population *t* years after the year 2000.



What are the maximum and minimum populations of the zebra?

Sketch this function.

**Example 3**

The top of a flagpole sways back and forth in the wind. The top sways 10 cm to the right (+10cm) and 10cm to the left (-10cm) and moves back and forth 240 times per minute. At t = 0 the flag pole is at its resting position (straight up), then began moving to the right.

Determine an equation to model the distance the top of the pole is from its resting position in terms of time (in seconds). (Hint: sketch the graph first).

**Example 4**

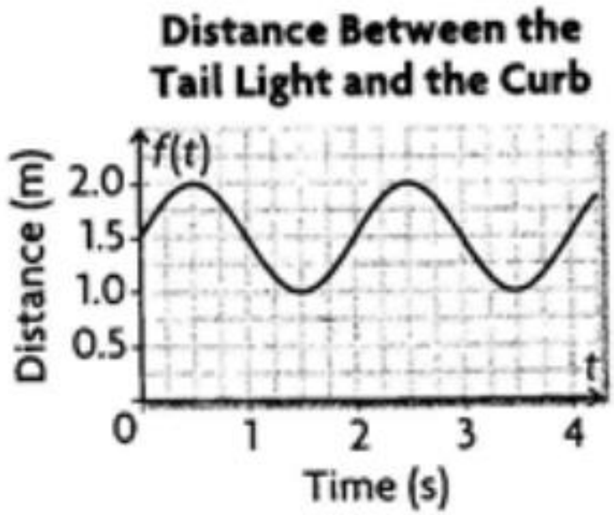
The average amount of daylight changes depending on the month of the year and can be modelled by a sinusoidal function. In one town, the month with the lowest average sunlight is December with 9 hours. The month with the highest amount of average daylight is June with 15 hours of daylight. Define a function that will predict the average amount of daylight, *D,* given the month of the year, *n.* (HINT: *n* is the number month of the year. It is easiest to make n=0 correspond to December, n=1 January and so on). Use your equation to predict the average amount of daylight in April.

MCF3M

# Assigned Problems

1. A ferris wheel has a radius of 10m and a lowest point 1m above the ground. The ferris wheel makes a complete revolution every 40 seconds. A person boards the ferris wheel at the wheel's lowest height. Define an equation that models the height of the rider given the time since they boarded the ferris wheel. Calculate a riders height after 55 seconds.

2. A load on a trailer has shifted causing the rear end of the trailer to swing left and right as shown in the graph below. (The graph shows the distance from the tail light on the trailer to the curb of the road)
   a) What is the equation of the axis and what does it represent?
   b) What is the amplitude of the graph? What does it represent?
   c) Find an equation for this graph.




3. The depth, $D$¸ in metres, of water in a harbour on a given day can be modeled using the function $D(t) = 6\sin 30t + 8$, where $t$ is the time past midnight, in hours.

   a) Sketch the graph of this function for one 24 hour day.

   b) Find the maximum and minimum depth of the water in the harbour. When do they occur?

   c) What is the period of this function? (include units)

   d) What is the equation of axis? What does it represent in this function?

4. The population of deer in a conservation area follow a sinusoidal model. In the year 2000 the deer population was at its lowest amount at 1200. In 2010 the deer population was at a maximum of 1600, before returning back to 1200 in 2020.

    a) What is the average deer population?
    b) Find an equation that models the deer population given the year.

**ANSWERS**

1. a) *h(t) = -10 cos(9t) + 11* b) about 18.1m

2. a) y = 1.5 m (distance from curb with no sway "resting position") b) 0.5 m (how far it sways)
c) D = 0.5 sin(180t) + 1.5

3. a) b) Maximum depth is 14 m and occurs at 3:00 (3AM), and 15:00 (3PM). Minimum depth is 2m and occurs at 9:00 (9AM) and 21:00 (9PM)
c) 12 hours d) y=8. The average depth in the harbor.

4. a) 1400 b) $P(n) = -200\cos(18n) + 1400$, where *n* is number of years since 2000.