Lesson 1 - Solutions to Examples.

1. $f(x) = x^4 - 8x^3 + 18x^2$

a) $\lim\limits_{x \to \pm\infty} f(x) = +\infty$ (even degree fcn with $a > 0$)

b) $f(0) = 0$ $\therefore$ $(0,0)$ is the only intercept.

$0 = x^4 - 8x^3 + 18x^2$
$0 = x^2(x^2 - 8x + 18)$
$x = 0 \qquad x^2 - 8x + 18 \neq 0$

c) critical points at $f'(x) = 0$

$f'(x) = 4x^3 - 24x^2 + 36x$

$f'(x) = 0 \rightarrow \qquad 0 = 4x^3 - 24x^2 + 36x$
$0 = 4x(x^2 - 6x + 9)$
$0 = 4x(x-3)^2$
$x = 0$ and $x = 3$

$f(0) = 0$
$f(3) = 3^4 - 8(3)^3 + 18(3)^2$
$f(3) = 27$

$\therefore$ $(0,0)$ and $(3,27)$ are critical points.

Looking at (0,0) $\Rightarrow$ checking y-values.

$f(-1) = (-1)^4 - 8(-1)^3 + 18(-1)^2$ $\quad$ $f(0) = 0$ $\quad$ $f(1) = 1^4 - 8(1) + 18(1)^2$

$f(-1) = 27$ $\qquad\qquad\qquad\qquad$ $f(1) = 11$

$\therefore$ (0,0) is a turning point. Since both $f(-1) > 0$ and $f(1) > 0$

local min.

Looking at (3, 27) $\rightarrow$ checking derivative.

$f'(1) = 4(1)^3 - 24(1)^2 + 36(1)$ $\quad$ $f'(3) = 0$ $\quad$ $f'(4) = 4(4)^3 - 24(4)^2 + 36(4)$

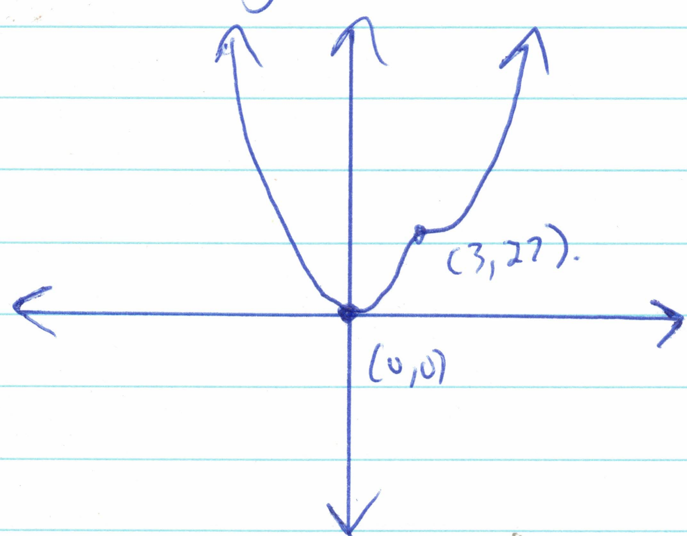
$f'(1) = 16$

$f'(1) > 0$ $\qquad\qquad\qquad\qquad$ $f'(4) = 304$

> 0

$\therefore$ (3, 27) is not a turning point since graph continues to increase. Must be a horizontal tangent and inflection point.

Sketch



2. $f(x) = 3x(x+2)^{\frac{2}{3}}$

$f'(x) = 3(x+2)^{\frac{2}{3}} + 3x\left(\frac{2}{3}\right)(x+2)^{-\frac{1}{3}}$

$f'(x) = 3(x+2)^{\frac{2}{3}} + 2x(x+2)^{-\frac{1}{3}}$

C.P. at $f'(x) = 0$

$0 = 3(x+2)^{\frac{2}{3}} + 2x(x+2)^{-\frac{1}{3}}$

$0 = (x+2)^{-\frac{1}{3}}\left(3(x+2) + 2x\right)$ factor out the $(x+2)^{-\frac{1}{3}}$

$0 = \dfrac{5x+6}{\sqrt[3]{x+2}}$ $\boxed{f'(x) = \dfrac{5x+6}{\sqrt[3]{x+2}}}$

$5x + 6 = 0$ but $f'(x)$ is undefined at

$x = -\frac{6}{5}$ $x = -2$ $\frac{5x+6}{0}$

$\therefore$ 2 critical points, one at $x = -\frac{6}{5}$, one at $x = -2$

bet you missed that one!

<u>check</u> $x = -2$

$f'(-2.1) > 0$ (increasing)

$f'(-2) = 0$ $\therefore$ $x = -2$ is a turning point and

$f'(-1.9) < 0$ (decreasing) local max.

check $x = -6/5$

$f'(-1) > 0$ (increasing)

$f'(-6/5) = 0$

$f'(-1.5) < 0$ (decreasing)

should have switched order.

~~or another local max~~

$\therefore$ $x = -6/5$ is a turning point (local min)

$\therefore$ $f(x)$ has 2 turning points!
(see graph in desmos).

3. $y = \dfrac{x^3}{x^2+1}$

$$\frac{dy}{dx} = \frac{3x^2(x^2+1) - x^3(2x)}{(x^2+1)^2}$$

$$\frac{dy}{dx} = \frac{x^4+3x^2}{(x^2+1)^2}$$

note that $(x^2+1)^2 \neq 0$

C.P. at $x^4+3x^2=0$

$x^2(x^2+3)=0$

$x=0 \quad x^2+3 \neq 0$

C.P. at $(0,0)$

check

$x=-1 \rightarrow y = \dfrac{(-1)^3}{(-1)^2+1} = -\frac{1}{2}$

$x=1 \rightarrow y = \dfrac{1^3}{1^2+1} = \frac{1}{2}$

graph goes from $-\frac{1}{2}$ to 0 to $\frac{1}{2}$ (continues to increase)

$\therefore$ $x=0$ not a turning point. There are none!