3. $f(x) = \dfrac{1}{x^2+1}$ or $f(x) = (x^2+1)^{-1}$

$$f'(x) = -(x^2+1)^{-2}(2x)$$

$$f'(x) = \frac{-2x}{(x^2+1)^2}$$

$f'(x) = 0 \rightarrow -2x = 0$ $\quad$ $f(0) = 1$

$x = 0$

$\therefore$ $(0,1)$ a critical point

$$f''(x) = \frac{-2(x^2+1)^2 - (-2x)(2)(x^2+1)(2x)}{(x^2+1)^4}$$

$$f''(x) = \frac{-2(x^2+1)^2 + 8x^2(x^2+1)}{(x^2+1)^4}$$

factor out
$\downarrow$ $-2(x^2+1)$

$$f''(x) = \frac{-2(x^2+1)\left[(x^2+1) - 4x^2\right]}{(x^2+1)^4}$$

$$f''(x) = \frac{-2(x^2+1)(-3x^2+1)}{(x^2+1)^4}$$

$$f''(x) = \frac{-2(-3x^2+1)}{(x^2+1)^3}$$

$$f''(x) = 0 \rightarrow -3x^2+1 = 0$$

$$x^2 = \frac{1}{3}$$

$$x = \pm\sqrt{\frac{1}{3}} \rightarrow x = \frac{\pm 1}{\sqrt{3}}$$

or $x = \pm\frac{\sqrt{3}}{3}$

since $x^2 = \frac{1}{3}$

$$f\left(\frac{\sqrt{3}}{3}\right) = \frac{1}{\frac{1}{3}+1} = \frac{1}{4/3} = \frac{3}{4} \qquad f\left(\frac{-\sqrt{3}}{3}\right) = \frac{1}{\frac{1}{3}+1} = \frac{3}{4}$$

<u>Note</u> $f''(0) \neq 0$ (that only happens at $\pm\frac{\sqrt{3}}{3}$).

In fact $f''(0) = \frac{-2(0+1)}{(0+1)^3}$

$$= -2$$

$$< 0 \quad \cap$$

$\therefore$ $(0,1)$ is a turning point and local max.

$\left(\frac{\sqrt{3}}{3}, \frac{3}{4}\right)$ and $\left(\frac{-\sqrt{3}}{3}, \frac{3}{4}\right)$ are both inflection points.

<u>graph</u> (rough!)