Unit 2 Review Solutions

1(b). $f(x) = \frac{x^3+4}{x^2}$

$f(x) = \frac{x}{x^2} + \frac{4}{x^2}$ ~~$\frac{x+4}{x^2}$~~

$f(x) = 0$

$x^3 + 4 = 0$

$x^3 = -4$

$x = \sqrt[3]{-4}$

VA at $x = 0$

Oblique asymptote at $y = x$

$\lim_{x \to +\infty} f(x) = +\infty$

$\lim_{x \to -\infty} f(x) = -\infty$

$f'(x) = \frac{3x^2(x^3) - (x^3+4)2x}{x^4}$

$f'(x) = \frac{x^4 - 8x}{x^4}$

$f'(x) = \frac{x^3 - 8}{x^3}$

$f'(x) = 0$

$x^3 - 8 = 0$

$x^3 = 8$

$x = 2$

$f(2) = \frac{2^3+4}{2^2} = 3$ (2,3)

$f''(x) = \frac{3x^2(x^3) - (x^3-4)(3x^2)}{x^6}$

$f''(x) = \frac{3x^5 - 3x^5 + 24x^2}{x^6}$

$f''(x) = \frac{24}{x^4}$

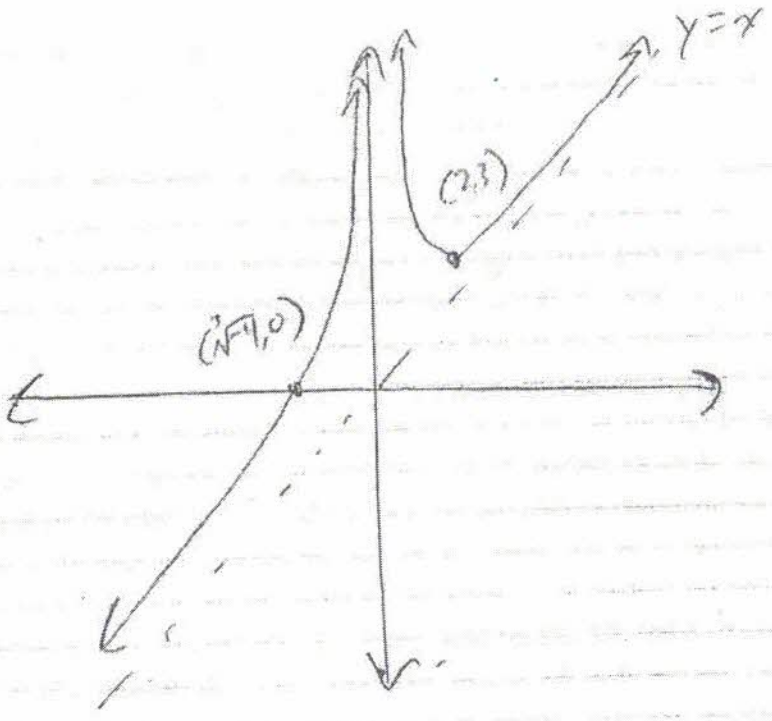
$f''(x) \neq 0$

so no I Ps.

(23) a I P

In fact $f''(x) > 0$

(concave up)


y=x
(2,3)
(-1,0)

b) $f(x) = \frac{6x^2 - 2}{x^3}$ ODD $f(-x) =$

V.A. at $x = 0$

$f(x) = 0 \quad 6x^2 - 2 = 0$

$6x^2 = 2$

$x^2 = \frac{1}{3}$

$x = \pm \frac{1}{\sqrt{3}}$

intercepts $\left(\frac{\pm 1}{\sqrt{3}}, 0\right)$

$$f'(x) = \frac{12x(x^3) - (6x^2 - 2)(3x^2)}{x^6}$$

$$f'(x) = \frac{-6x^4 + 6x^2}{x^6}$$

$$f'(x) = \frac{-6x^2 + 6}{x^4}$$

$f'(x) = 0$

$-6x^2 + 6 = 0$

$-6x^2 = -6$

$x^2 = 1$

$x = \pm 1$

$$f''(x) = \frac{-12x(x^4) - (-6x^2 + 6)4x^3}{x^8}$$

$$f''(x) = \frac{12x^5 - 24x^3}{x^8}$$

$$f''(x) = \frac{12x^2 - 24}{x^5}$$

$f(\pm 1) = \pm 4$

$(1, 4) \quad (-1, -4)$

$f''(x) = 0$

$12x^2 - 24 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

$$f(\pm\sqrt{2}) = \pm\frac{10}{\sqrt{8}} = \pm\frac{10}{2\sqrt{2}} = \pm\frac{5}{\sqrt{2}} = \pm\frac{5\sqrt{2}}{2}$$

∴ turning points $(1,4)$ and $(-1,-4)$

inflection points $(\sqrt{2}, \frac{5\sqrt{2}}{2})$ $(-\sqrt{2}, \frac{-5\sqrt{2}}{2})$

$$\lim_{x\to+\infty} \frac{6x^2-2}{x^3}$$

$$= \lim_{x\to+\infty} \frac{\frac{6}{x}-\frac{2}{x^3}}{1}$$

$$= 0^+$$

∴ $\lim_{x\to-\infty} f(x) = 0^-$

(odd)