Introduction to Exponents & Some Basic Exponent Laws

exponent

Here is an example of a power: $\boxed{3^4}$ $= 3 \times 3 \times 3 \times 3$

base $= 81$

Notice the pattern if we multiply powers of the same base:

$5^4 \times 5^2$
$= 5 \times 5 \times 5 \times 5 \times 5 \times 5$
$= 5^6$

$2^5 \times 2^2$
$= 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 2^6$

$x^2(x^3)$
$= x \cdot x \cdot x \cdot x \cdot x$
$= x^5$

The multiplication law: $a^m \times a^n = a^{m+n}$

To multiply powers of LIKE BASES we add the exponents.

Examples: $2^8(2^3)$
$= 2^{11}$
$256(8)$
$=$

$y^4(y^{10})$
$= y^{14}$

Notice the pattern if we divide powers of the same base:

$5^4 \div 5^2 = \frac{5^4}{5^2}$

$= \frac{\cancel{5} \times \cancel{5} \times 5 \times 5}{\cancel{5} \times \cancel{5}}$

$= 5 \times 5$

$= 5^2$

$\frac{x^5}{x^4} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}}$

$= x^1$

The division law: $a^m \div a^n = a^{m-n}$ OR $\frac{a^m}{a^n} = a^{m-n}$

To divide powers of LIKE BASES we *subtract* the exponents

Examples:

$$\frac{5^{11}}{5^8}$$

$$= 5^3$$

$$\frac{y^2(y^6)}{y^4}$$

$$= \frac{y^8}{y^4} = y^4$$

The examples below illustrate the idea of "power of a power" or "power on a power"

$\left(10^4\right)^2$

$= 10^4 \times 10^4$

$= 10^8$

$= 100,000,000$

$\left(5^2\right)^3$

$= 5^2 \times 5^2 \times 5^2$

$= 5^6$

The power on a power law: $\left(a^m\right)^n = a^{mn}$

To simplify a power on a power we multiply the exponents.

Examples:

$\left(2^2\right)^5$

$= 2^{10}$

$= 4^5$

$\left(x^2\right)^9$

$= x^{18}$

The following examples illustrate "power of a product"

$(2 \times 3)^3 = 6^3$

$= (2\times 3)\times(2\times 3)\times(2\times 3)$

$= 2\times 2\times 2\times 3\times 3\times 3$

$= 2^3 \times 3^3$

$(2x)^4$

$= (2x)(2x)(2x)(2x)$

$= 2^4x^4 = 16x^4$

The power of a product rule can be summarized as follows: $(ab)^n = a^n b^n$

Examples

$(2x)^7$

$= 2^7x^7$

$= 128x^7$

$(-3x^2)^4$

$= (-3)^4(x^2)^4$

$= 81x^8$

$(ab)^n = a^n b^n$

$(a+b)^n \neq a^n + b^n$

$(2+x)^2 \neq 2^2 + x^2$

$(2+x)(2+x)$

$= 4 + 4x + x^2$

The examples below look at powers with rational bases.

$\left(\frac{3}{4}\right)^3$

$= \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4}$

$= \frac{3^4}{4^4} = \frac{81}{256}$

$\left(\frac{1}{5}\right)^4$

$= \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$

$= \frac{1^4}{5^4} = \frac{1}{5^4}$

This rule can be summarized as follows: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Examples:

$\left(\frac{2}{9}\right)^2$

$= \frac{2^2}{9^2}$

$= 4/81$

$\left(\frac{-1}{4}\right)^3$

$= \frac{(-1)^3}{4^3}$

$= -1/64$

$\left(\frac{-1}{2}\right)^4$

$= \frac{(-1)^4}{2^4}$

$= 1/16$

Summary of Basic Exponent Laws

Multiplication of Powers	$a^m \times a^n = a^{m+n}$
Division of Powers	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Power	$(a^m)^n = a^{mn}$
Power of a Product	$(ab)^n = a^n b^n$
Power of a Quotient/Powers with Rational Bases	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Examples

Simplify each of the following:

$$\frac{x^2(x^3)^5 x}{x^{10}}$$

$$= \frac{x^2 \cdot x^{15} x^1}{x^{10}}$$

$$= \frac{x^{18}}{x^{10}} = x^8$$

$$\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^1$$

$$= \left(\frac{2}{3}\right)^5$$

$$= \frac{2^5}{3^5} *$$

$$= \frac{32}{243}$$

$$\frac{(2x^2)^3}{4x^3}$$

$$= \frac{2^3 x^6}{4x^3}$$

$$= \frac{8x^6}{4x^3}$$

$$= 2x^3$$