

## Zero and Negative Exponents

MCF3M

$$2^4 = 2 \times 2 \times 2 \times 2$$

$$2^1 = 2$$

$$2^0 = ?$$

$$2^{-1} = ?$$

$$2^{-3} = ?$$

$2^{-3}$	$1/8$	
$2^{-2}$	$1/4$	●
$2^{-1}$	$1/2$	●
$2^0$	$1 \times 2$	●
$2^1$	$2 \times 2$	● ●
$2^2$	$4 \times 2$	● ● ● ●
$2^3$	$8 \times 2$	● ● ● ● ● ● ● ●
$2^4$	$16 \times 2$	● ● ● ● ● ● ● ● ● ● ● ● ● ● ● ●

Handwritten notes on the table: A large bracket on the left side groups the rows from  $2^{-3}$  to  $2^0$ . To the left of the table, three arrows labeled  $\div 2$  point upwards from  $2^1$ ,  $2^2$ , and  $2^3$  to  $2^0$ ,  $2^{-1}$ , and  $2^{-2}$  respectively. In the  $2^3$  row, the 5th dot from the left is highlighted in yellow.

First lesson is that:  $2^0 = 1$

or more general:

$$a^0 = 1$$

Second lesson is that  $2^{-n} = \frac{1}{2^n}$

$$2^{-3} = \frac{1}{2^3}$$

or more general:

$$a^{-n} = \frac{1}{a^n}$$

$$4^0 = 1 \quad 4^3 = 1 \times 4 \times 1 \times 4$$

Examples: evaluate the following:

$$7^0 = 1$$

$$4^{-3} = \frac{1}{4^3}$$

$$= \frac{1}{64}$$

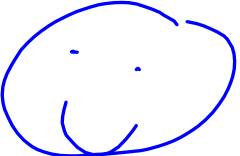
$$4^{-3} = 1 \div 4 \div 4 \div 4$$

Notice that the rules above allow our exponent laws to work. Look at the following:

$$\frac{5^3}{5^3} = 5^0 = 1$$

$$\text{or } \frac{125}{125} = 1$$

$$\frac{3^3}{3^7} = 3^{-4} = \frac{1}{3^4}$$

$$= \frac{\cancel{3} \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3} \times \cancel{3} \times 3 \times 3 \times 3 \times 3} = \frac{1}{3^4}$$


What about if your base is a fraction?

$$a^{-n} = \frac{1}{a^n}$$

Example

$$\begin{aligned} & \left(\frac{2}{3}\right)^{-2} \\ &= \frac{1}{\left(\frac{2}{3}\right)^2} \\ &= \frac{1}{\frac{2^2}{3^2}} \end{aligned} \quad \begin{aligned} &= 1 \div \frac{2^2}{3^2} \\ &= \frac{1}{1} \times \frac{3^2}{2^2} \\ &= \frac{3^2}{2^2} = \left(\frac{3}{2}\right)^2 \end{aligned}$$

In general:  $\left(\frac{a}{b}\right)^{-n} =$

$$\begin{aligned} & \left(\frac{b}{a}\right)^n \\ &= \frac{b^n}{a^n} \end{aligned}$$

In summary:

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

$$3^{-2} = \frac{1}{3^2}$$

$$\left(\frac{2}{9}\right)^{-3} = \left(\frac{9}{2}\right)^3$$

Examples

$$5^{-1}(5^3)(5^{-4})$$

$$\begin{aligned} &= 5^{-2} \\ &= \frac{1}{5^2} * \\ &= \frac{1}{25} \end{aligned}$$

$$\left(\frac{-1}{3}\right)^{-4}$$

$$\begin{aligned} &= \left(\frac{-3}{1}\right)^4 \\ &= (-3)^4 \\ &= 81 \end{aligned}$$

$$\frac{x^2(x^5)^{-1}x}{x^{-2}}$$

$$\begin{aligned} &= \frac{x^2 x^{-5} x^1}{x^{-3}} \\ &= \frac{x^{-2}}{x^{-3}} \\ &= x^1 \quad \begin{array}{l} -2 - (-3) \\ = -2 + 3 \\ = 1 \end{array} \\ &= x \end{aligned}$$

