

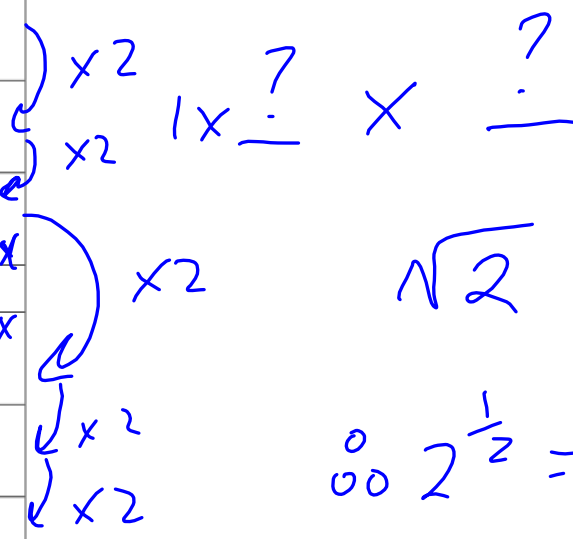
Rational (aka "Fraction") Exponents

$2^{\frac{1}{2}} = ?$ $5^{\frac{1}{3}} = ?$

$2^0 = 1$
 $2^{-2} = \frac{1}{4}$

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2^{-3}	$\frac{1}{8}$
2^{-2}	$\frac{1}{4}$
2^{-1}	$\frac{1}{2}$
2^0	1
$2^{\frac{1}{2}}$	$\sqrt{2}$
2^1	2
2^2	4
2^3	8
2^4	16



$\sqrt{2}$
 $\therefore 2^{\frac{1}{2}} = \sqrt{2}$

● $9^{\frac{1}{2}} = ?$

9^{-2}	$\frac{1}{81}$	
9^{-1}	$\frac{1}{9}$	$\times 9$
9^0	1	$\times 9$
$9^{\frac{1}{2}}$	3	$\times 9$
9^1	9	$\times 9$
9^2	81	$\times 9$

4^{-2}	$\frac{1}{16}$	
4^{-1}	$\frac{1}{4}$	
4^0	1	$\times 2$
$4^{\frac{1}{2}}$	2	$\times 2$
4^1	4	$\times 4$
4^2	16	$\times 4$

In general $a^{\frac{1}{2}} = \sqrt{a}$

But what about $9^{\frac{1}{3}}$? And $2^{\frac{1}{4}}$ and so on?

Exponent laws can help as well.

$$4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^1$$

$$\therefore 4^{\frac{1}{2}} = 2 \quad \sqrt{4}$$

$$5^{\frac{1}{2}} \times 5^{\frac{1}{2}} = 5^1$$

$$\therefore 5^{\frac{1}{2}} = \sqrt{5}$$

$$8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8^1$$

$$\therefore 8^{\frac{1}{3}} = 2 \quad \sqrt[3]{8} = 2$$

$$_ \times _ \times _ = 8$$

$$32^{\frac{1}{5}} \times 32^{\frac{1}{5}} \times 32^{\frac{1}{5}} \times 32^{\frac{1}{5}} \times 32^{\frac{1}{5}} = 32^1$$

$$\therefore 32^{\frac{1}{5}} = 2 \quad \sqrt[5]{32} = 2$$

$$2^5 = 32$$

In general: $a^{\frac{1}{n}} = \sqrt[n]{a}$ | $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{10}$

Examples: Evaluate the following:

$$81^{\frac{1}{2}} = \sqrt{81} = 9$$

$$16^{\frac{1}{4}} = \sqrt[4]{16} = 2$$

$25^{\frac{1}{3}} = \sqrt[3]{25} \approx 2.92$ (radical form)
 $16^{\frac{1}{2}} = \frac{1}{16^{\frac{1}{2}}} = \frac{1}{\sqrt{16}} = \frac{1}{4}$ (exponent form)

$$81^0 = 1$$

How might we evaluate the following?

$$81^{\frac{3}{4}} = \left(81^{\frac{1}{4}}\right)^3 = \left(\sqrt[4]{81}\right)^3 = (3)^3 = 27$$

$$8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$$

In general: $a^{\frac{m}{n}} = \left(\sqrt[n]{a} \right)^m$ or $\sqrt[m]{a^n}$

More examples.

Evaluate each of the following:

$$= \left(\sqrt[3]{15} \right)^2 \quad \text{or} \quad \sqrt[3]{15^2} = \sqrt[3]{225}$$

$$= \sqrt[3]{225} \approx 6.08$$

$$\left(\frac{9}{16} \right)^{\frac{3}{2}} = \frac{9^{\frac{3}{2}}}{16^{\frac{3}{2}}}$$

$$= \frac{(\sqrt{9})^3}{(\sqrt{16})^3} = \frac{3^3}{4^3} = \frac{27}{64}$$

$\left(\frac{a}{b} \right)^n = \frac{a^n}{b^n}$

$$\begin{aligned} &= 25^{\frac{3}{2}} \\ &= (\sqrt{25})^3 \\ &= 5^3 \\ &= 125 \end{aligned}$$

$$\begin{aligned} &= (-27)^{\frac{2}{3}} \\ &= (\sqrt[3]{-27})^2 \\ &= (-3)^2 \\ &= 9 \end{aligned}$$