$2^0 = 1$

$2^{-2} = \frac{1}{4}$

Rational (aka "Fraction") Exponents

MCF3M

$2^{\frac{1}{2}} = ?$ $5^{\frac{1}{3}} = ?$

2^{-3}	$\frac{1}{8}$
2^{-2}	$\frac{1}{4}$
2^{-1}	$\frac{1}{2}$
2^0	1
$2^{\frac{1}{2}}$	$\sqrt{2}$
2^1	2
2^2	4
2^3	8
2^4	16

x2 x2 x2 x2 x2

$1 \times ? \times ?$

$\sqrt{2}$

$\therefore 2^{\frac{1}{2}} = \sqrt{2}$

$9^{\frac{1}{2}} = ?$

9^{-2}	$\frac{1}{81}$
9^{-1}	$\frac{1}{9}$
9^0	1
$9^{\frac{1}{2}}$	3
9^1	9
9^2	81

4^{-2}	$\frac{1}{16}$
4^{-1}	$\frac{1}{4}$
4^0	1
$4^{\frac{1}{2}}$	2
4^1	4
4^2	16

In general $a^{\frac{1}{2}} = \sqrt{a}$

But what about $9^{\frac{1}{3}}$? And $2^{\frac{1}{4}}$ and so on?

Exponent laws can help as well.

$4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^1$

$\therefore 4^{\frac{1}{2}} = 2$ $\quad \sqrt{4}$

$5^{\frac{1}{2}} \times 5^{\frac{1}{2}} = 5^1$

$\therefore 5^{\frac{1}{2}} = \sqrt{5}$

$8^{\frac{1}{3}} \times 8^{\frac{1}{3}} \times 8^{\frac{1}{3}} = 8^1$ $\quad \_ \times \_ \times \_ = 8$

$\therefore 8^{\frac{1}{3}} = 2$ $\quad \sqrt[3]{8} = 2$

$32^{\frac{1}{5}} \times 32^{\frac{1}{5}} \times 32^{\frac{1}{5}} \times 32^{\frac{1}{5}} \times 32^{\frac{1}{5}} = 32^1$

$\therefore 32^{\frac{1}{5}} = 2$ $\quad \sqrt[5]{32} = 2$

$2^5 = 32$

In general: $a^{\frac{1}{n}} = \sqrt[n]{a}$

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{10}$

Examples: Evaluate the following:

$81^{\frac{1}{2}}$

$= \sqrt[2]{81}$

$= 9$

$16^{\frac{1}{4}}$

$= \sqrt[4]{16}$

$= 2$

$25^{\frac{1}{3}}$

$= \sqrt[3]{25}$

≈ 2.92

exponent form

radical form

$2^{-3} = \frac{1}{2^3}$

$16^{-\frac{1}{2}} = \frac{1}{16^{\frac{1}{2}}}$

$= \frac{1}{\sqrt{16}}$

$= \frac{1}{4}$

$81^0 = 1$

81^1

How might we evaluate the following?

$81^{\frac{3}{4}}$

$= \left(81^{\frac{1}{4}}\right)^3$

$= \left(\sqrt[4]{81}\right)^3$

$= (3)^3$

$= 27$

$8^{\frac{2}{3}}$

$= \left(8^{\frac{1}{3}}\right)^2$

$= \left(\sqrt[3]{8}\right)^2$

$= 2^2 = 4$

In general: $a^{\frac{m}{n}} = \left(\sqrt[m]{a}\right)^n$ or $\sqrt[m]{a^n}$

More examples.

Evaluate each of the following:

$15^{\frac{2}{3}}$

$= \left(\sqrt[3]{15}\right)^2$ or $\sqrt[3]{15^2} = \sqrt[3]{225}$

$= \sqrt[3]{225} \doteq 6.08$

$\left(\frac{9}{16}\right)^{\frac{3}{2}}$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$= \frac{9^{\frac{3}{2}}}{16^{\frac{3}{2}}}$

$= \frac{\left(\sqrt{9}\right)^3}{\left(\sqrt{16}\right)^3} = \frac{3^3}{4^3} = \frac{27}{64}$

$25^{\frac{3}{2}}$

$= \left(\sqrt{25}\right)^3$

$= 5^3$

$= 125$

$(-27)^{\frac{2}{3}}$

$= \left(\sqrt[3]{-27}\right)^2$

$= (-3)^2$

$= 9$

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