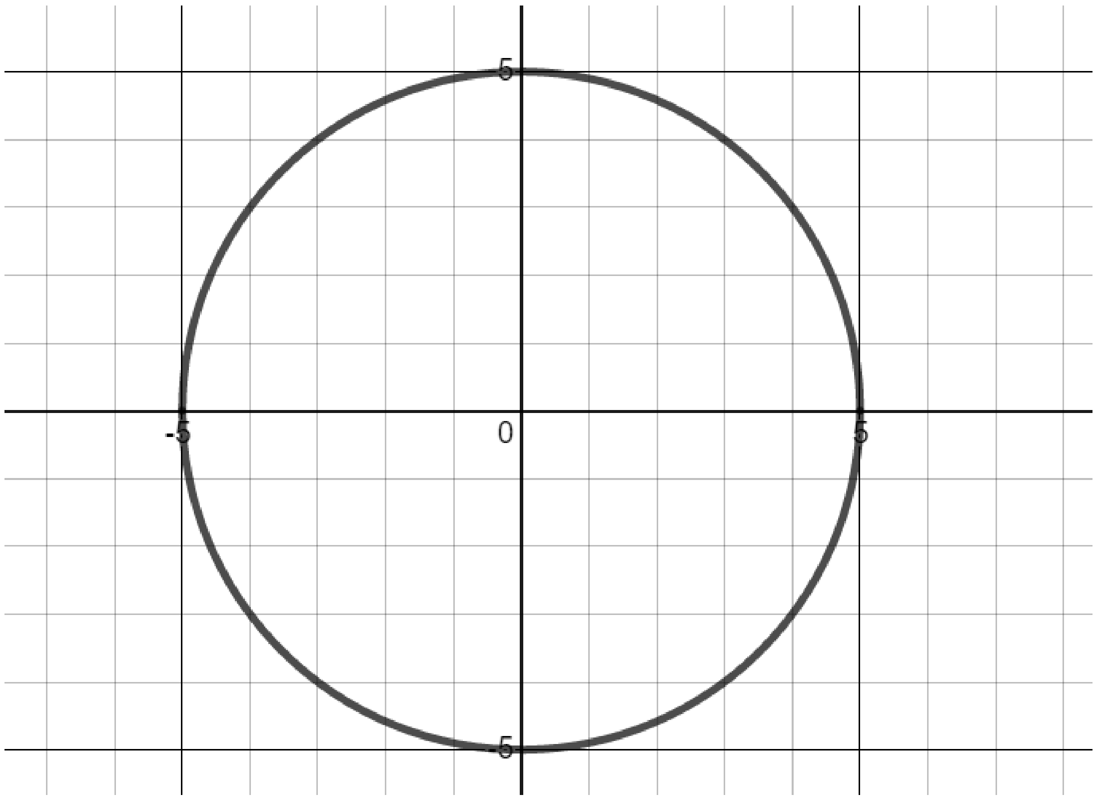
Implicit Differentiation

Consider the example of finding the derivative of a circle.



How do we take the derivative of an equation defined *implicitly?*

Equations Defined Explicitly: (Think "clearly defined")

Equations are defined explicitly are written in the form y = f(x) and are the only type we have dealt with so far in this course.
Examples: $y = x^2$, $y = \sqrt{x^2} - 6$ $y = 3x + 1$ etc.

Equations Defined Implicitly:

Equations can be defined implicitly (y is not given explicitly in terms of x).

Examples: $x^2 + y^2 = 25$, $y^5 + x^2y - 2x^2 = -1$, etc..

Equations like the ones above may or may not be rewritten explicitly. Often they are not functions.

Implicit equations have instantaneous rate of change (derivatives) and tangent lines, so we want to be able to differentiate them as well. This process is called implicit differentiation.

Examples

Find $\frac{dy}{dx}$ for the circle defined by the equation above. Find the equation of the tangent to the circle at the point $(3,4)$.

Find $\frac{dy}{dx}$ for the curved defined by $\frac{x^2}{4} + y^2 = 1$

Sometimes implicit differentiation is easier even if the function is defined explicitly.

Example: Find the slope of the tangent to the curve $y = \sqrt[3]{x^2 + 4}$ at the point where x = 2.

Implicit differentiation can make finding a derivative easier. However, for equation such as $y^5 + x^2 y - 2x^2 = -1$, it is the only way to find a derivative (this equation cannot be written explicitly).