The Chain Rule

Consider taking the derivative of the function:

$$y = (x^2 - 2x)^4$$

There really is no such rule as the "power of a function" rule. It is actually the power rule used with what is referred to as the ***chain rule.***

The chain rule is as follows:

If *y* is a function of *u* and *u* is a function of *x* then:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

We can rewrite the example above and show how the chain rule (with the power rule) gives us the derivative:

In fact, we can combine the power rule with the chain rule and write:

We can also use the chain rule multiple times, with any variable names we like.

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

$$\frac{dz}{dx} = \frac{dz}{dy} \times \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{dy}{dr} \times \frac{du}{dx}$$

$$\frac{dx}{ds} = \frac{ds}{dt} \times \frac{dt}{dx}$$

The chain rule is more difficult to understand in prime notation. But it is defined as:

If $h(x) = f[g(x)]$ then $h'(x) = f'[g(x)]g'(x)$

The chain rule does not immediately give us any new functions we can take the derivative of. (Since we have already been using the chain rule without knowing it).

$f(x) = \sqrt{3x^2 - x}$

We could use the chain rule to simplify the following problem:

Find the slope of the tangent to the graph of

$y = \left[4(3-x^2) + \frac{3}{(3-x^2)}\right]^5 + \left[4(3-x^2) + \frac{3}{(3-x^2)}\right]^3$ at x = 2

Here is a new derivative rule for you: $\frac{d}{dx}(\sin x) = \cos x$

The chain rule will now allow us to find derivatives of all sorts of trigonometric functions:

$y = \sin(3x^2 - 4x)$

$f(x) = 10 \sin \sqrt{x}$

Can we prove the chain rule?

Let *y* be a function of *u* and *u* be a function of *x*:

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