3. a) $f(x) = \sin(x^2 - 5x)$

$f'(x) = \cos(x^2 - 5x)\ (2x - 5)$

$f'(x) = (2x - 5)\cos(x^2 - 5x)$

b) $y = \sin^2(5x)$

or $y = [\sin(5x)]^2$

$\frac{dy}{dx} = 2\sin(5x)\ (\cos(5x))\ (5)$

$\frac{dy}{dx} = 10\sin(5x)\cos(5x)$

4. $f(x) = \frac{1}{x^2 - 2x + 2}$ or $f(x) = (x^2 - 2x + 2)^{-1}$

$f'(x) = -(x^2 - 2x + 2)^{-2}\ (2x - 2)$

$f'(x) = \frac{-2x + 2}{(x^2 - 2x + 2)^2}$

$f'(x) = 0 \rightarrow -2x + 2 = 0$

$-2x = -2$

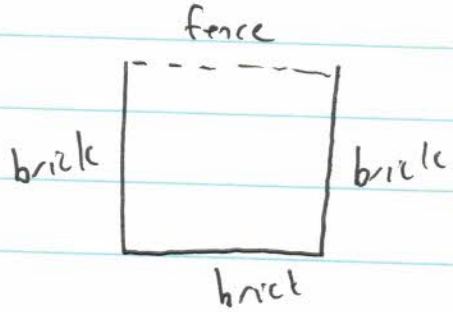
$x = 1$

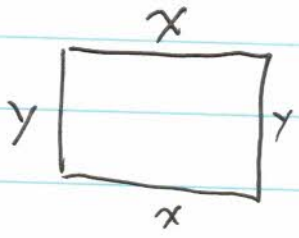
<u>check</u> $f(0) = \frac{1}{2}$ $f(1) = 1$

$f(3) = \frac{1}{5}$

& max value is 1 and min value is $\frac{1}{5}$.

5.





$A = xy$

$1000 = xy$

$y = \frac{1000}{x}$

Let $C = \text{cost}(\$)$

$C = 192x + 192y + 192y + 48x$

$C = 240x + 384y$

but $y = \frac{1000}{x}$

$C = 240x + 384\left(\frac{1000}{x}\right)$

$C = 240x + \frac{384000}{x}$

$\frac{dC}{dx} = 240 - \frac{384000}{x^2}$

critical points at $\frac{dC}{dx} = 0$

$$0 = 240 - \frac{384\,000}{x^2}$$

$$\frac{384\,000}{x^2} = 240$$

$$x^2 = \frac{384\,000}{240}$$

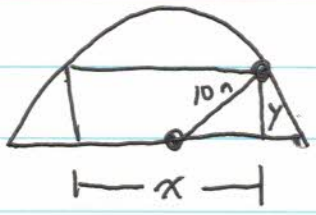
$$x = \sqrt{\frac{384000}{240}}$$

$$x = \sqrt{1600} \qquad x = 40 \qquad y = \frac{1000}{40}$$

$$y = 25$$

Make side with fence 40m.
Other 2 sides 25m.

6.



$10^2 = \left(\frac{x}{2}\right)^2 + y^2$

$100 = \frac{x^2}{4} + y^2$

$A = xy$

$y^2 = 100 - \frac{x^2}{4}$

$y = \sqrt{100 - \frac{x^2}{4}}$

$A = x\,\frac{\sqrt{400-x^2}}{2}$

$y = \sqrt{\frac{400-x^2}{4}}$

$A = \frac{1}{2}x\sqrt{400-x^2}$

$y = \frac{\sqrt{400-x^2}}{2}$

$\frac{dA}{dx} = \frac{1}{2}\left(\sqrt{400-x^2}\right) + \frac{1}{2}x \cdot \frac{1}{2}(400-x^2)^{-1/2}(-2x)$

$\frac{dA}{dx} = \frac{1}{2}\sqrt{400-x^2} - \frac{1}{2}x^2(400-x^2)^{-1/2}$

$\frac{dA}{dx} = \frac{\sqrt{400-x^2}}{2} - \frac{x^2}{2\sqrt{400-x^2}}$

$\frac{dA}{dx} = \frac{400-x^2-x^2}{2\sqrt{400-x^2}} = \frac{400-2x^2}{2\sqrt{400-x^2}}$

$\frac{dA}{dx} = 0 \rightarrow \quad 0 = 400 - 2x^2$

$2x^2 = 400$

$x^2 = 200$

$x = \sqrt{200}$

$x = 10\sqrt{2}$

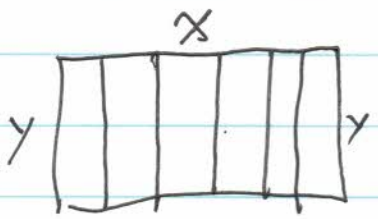
$y = \frac{\sqrt{400 - x^2}}{2}$

$y = \frac{\sqrt{400 - 200}}{2}$

$y = \frac{\sqrt{200}}{2} = 5\sqrt{2}$

∴ rectangle with max area is $10\sqrt{2}$ m by $5\sqrt{2}$ m.

7.



$390 = xy$

$y = \frac{390}{x}$

$P = 2x + 7y$

$P = 2x + 7\left(\frac{390}{x}\right)$

$P = 2x + \frac{2730}{x}$

$\frac{dP}{dx} = 2 - \frac{2730}{x^2}$

C.P. at $0 = 2 - \frac{2730}{x^2}$

$\frac{2730}{x^2} = 2$

$x^2 = \frac{2730}{2}$

$x^2 = 1365$

$\rightarrow x = \sqrt{1365}$

$x \doteq 36.9\text{cm.}$

$y = \frac{390}{\sqrt{1365}}$

$y \doteq 10.6\text{cm.}$