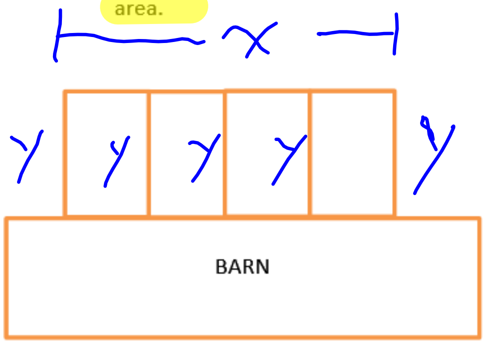
Optimization Problems

Example 1

Keith owns a dog kennel. He wants to create a fenced in area along the side of a barn for four dogs. Each dog will be separated by fencing, and the barns will serve as one of sides of the fenced in area. He has exactly 200 feet of fencing to work with. Find the dimensions that will maximize the total fenced in area.

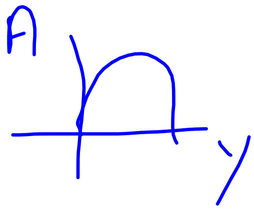


$$A = xy \qquad 200 = x + 5y$$

$$x = 200 - 5y$$

$$A = (200 - 5y)y$$

$$A = 200y - 5y^2 \quad *$$



$$\frac{dA}{dy} = 200 - 10y$$

$$\frac{dA}{dy} = 0 = 200 - 10y$$

$$10y = 200$$

$$y = 20 \text{ ft}$$

$$x = 200 - 5(20)$$

$$x = 100 \text{ ft}$$

Example 2

A can of corn is made to hold 540mL of corn. Find the dimensions of the can that will minimize the surface area of the can.

base/top

lateral surface

$V = 540\text{ cm}^3$

$V = \pi r^2 h$

$540 = \pi r^2 h$

$h = \dfrac{540}{\pi r^2}$

$SA = 2\pi r^2 + 2\pi r h$

$SA = 2\pi r^2 + 2\pi r\left(\dfrac{540}{\pi r^2}\right)$

$SA = 2\pi r^2 + \dfrac{1080}{r}$

$\dfrac{dSA}{dr} = 4\pi r - \dfrac{1080}{r^2}$

$1080r^{-1}$

$-1080r^{-2}$

h

$0 = 4\pi r - \dfrac{1080}{r^2}$

$4\pi r = \dfrac{1080}{r^2}$

$4\pi r^3 = 1080$

$r = \sqrt[3]{\dfrac{1080}{4\pi}}$

$r = \sqrt[3]{\dfrac{270}{\pi}}$

$\boxed{r \doteq 4.41\text{ cm}}$

$h \doteq \dfrac{540}{\pi(4.41)^2}$

$\boxed{h \doteq 8.8\text{ cm}}$

Example 3

Frozen concentrated juice is packaged in a cylinder. The lateral surface is made from cardboard and the base and top from aluminum. The aluminum is 3 times more expensive than the cardboard. If the juice container has a volume of 350 cm^3, then ***find the dimensions*** of the cylinder that will minimize the cost of the material used

$$V = \pi r^2 h$$

$$350 = \pi r^2 h$$

$$h = \frac{350}{\pi r^2}$$

$$SA = 2\pi r^2 + 2\pi r h$$

Let k = cost per cm^3 of lateral surface

$$C = 2\pi r^2(3k) + 2\pi r h(k)$$

$$C = 6k\pi r^2 + 2k\pi r h$$

$$C = 6k\pi r^2 + 2k\pi r\left(\frac{350}{\pi r^2}\right)$$

$$C = 6k\pi r^2 + \frac{700k}{r}$$

$$\frac{dC}{dr} = 12k\pi r - \frac{700k}{r^2}$$

$$0 = 12k\pi r - \frac{700k}{r^2}$$

$$\frac{700\cancel{k}}{r^2} = 12\cancel{k}\pi r$$

$$12r^3 = 700$$

$$r = \sqrt[3]{\frac{700}{12}}$$

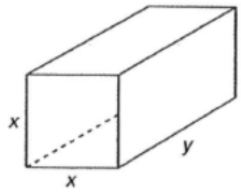
$$r \doteq 3.88\text{cm}$$

$$h = \frac{350}{\pi(3.88)^2}$$

$$h \doteq 7.4\text{cm}$$

Assigned Problems

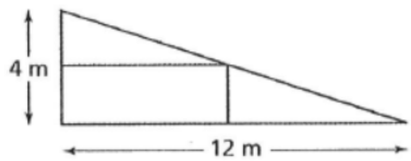
1. A net enclosure for practising golf shots is open at one end, as shown. Find the dimensions that minimize the amount of netting and that give a volume of 144 m^3. (Netting is required only on the sides, the top, and the far end.)



2. **Application**: Mark will make an open rectangular box with a square base from two different materials. The material for the bottom costs $\$3/m^2$. The material for the sides costs $\$2/m^2$. Find the dimensions of the box with the maximum volume that Mark can make for \$120.

3. The volume of a rectangular box will be 15 m^3. The box has a square base and top. The material for the base costs $\$7.50/m^2$. The material for the top costs $\$2.50/m^2$ and for the sides $\$4.50/m^2$. Find the dimensions that will minimize the cost of materials for the box.

4. The right triangle represents a lot with the dimensions shown. Doug wants to build and fence a rectangular dog kennel inside this lot. What is the maximum possible area for the kennel?



5. Text page 141 #3, 4, 13,

ANSWERS

1. 6m by 6m by 4m **2.** Base has side length $\frac{2\sqrt{30}}{3}$m and height is $\frac{\sqrt{30}}{2}$ (or approx. 3.65m and 2.74m)

3. base is side length is approx. 2.38m (or $\frac{3}{\sqrt[3]{2}}$) and height approx. 2.65m. **4.** 12m^2