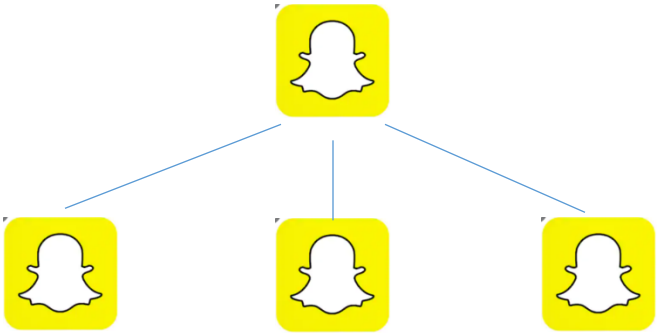
Exploring Exponential Growth

Example: Snapchat Party! Suppose you decide to have a party. You send a message through Snapchat to your 3 best friends. Every day each one of those friends passes the invitation on to 3 more friends and so on and so on…

1. Complete the tree below to show the number of invitations sent per day. Note that day zero corresponds to the first invitation before it has been sent.

Day 0



Day 1

Day 2

Day 3

2. Complete the table below.

Day	Number of Invites Sent that Day
0	1
1	3
2	
3	
4	
5	

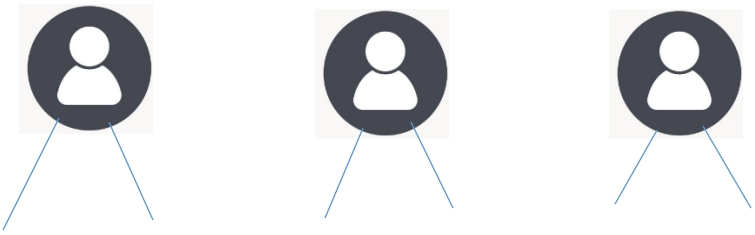
3. If this pattern continues, can you determine how many invitations will be sent on the 10^{th} day?

Example – A Contagious Disease

Suppose that 3 people in a town have a contagious disease. Every week, each of those people infects 2 other people. Those people continue the pattern of spreading the disease to two other people and so on.

1. Complete the tree below.

Week 0



Week 1

Week 2

2. Use the pattern to complete the table below.

Week	Number of New Infections that Week
0	3
1	6
2	
3	
4	
5	

Example – Grocery Store Job

Suppose that you start a job working at the grocery store. The job pays $15.25 per hour. At the end of each year you receive a 4% raise.

1. After working for one year, what is your new hourly wage?

In question #1 above there are 2 methods to find the new hourly wage:

Method #1 – $4\%\ of\ \$15.25 = \ 0.04\ \times \$15.25 = \$0.61$

$\$0.61 + \$15.25 = \$\mathbf{15.86}$

Method #2 - $\$15.25 \times 1.04 = \$\mathbf{15.86}$

Both methods give the correct answer of $15.86. The second method works since you are basically saying you want to calculate 104% of $15.25. (100% of your pay plus 4%). The second method is obviously quicker and is important to understand for this topic.

2. Use multiplying by 1.04 to complete the table below:

Year	Hourly wage
0	$15.25
1	$15.86
2	
3	
4	
5	

3. Can you calculate how much you would make working at the grocery store if you worked for *20 years*? (you can come back to this problem later if you are stuck).

Example – Bald Eagle Population

The population of Bald Eagles has made a large rebound in North America.

Suppose that in 1990 there were only 3000 nesting Bald Eagle pairs in America.

However, each year after that the number of nesting Bald Eagles increased by 6.8% each year.

1. What number could you multiply 3000 by to get the number of nesting pairs of Bald Eagles in 1991? (Check your solution in the table below).

2. Complete the table below.

Year	Bald Eagle Nesting Pairs
1990	3000
1991	3204
1992	
1993	
1994	
1995	
1996	
1997	

3. Can you calculate the number of Bald Eagle nesting pairs in the year 2015? (If you are stuck come back to this later).

Exploring Exponential Decay

Example – Car Depreciation

Depreciation is the decline in a car's value over the course of its useful life.

A 2018 BMW M2 is valued at $65 000 and depreciates on average 20% per year.

1. Since the car is depreciating at 20% per year, the reaming value at the end of the first year is _____% of the original value.

Therefore to find the depreciated value, multiply the previous year's value by _____.

2. Complete the table below to calculate the value of the car at the end of each of the first five years of ownership.

Year End	Value of Car ($)
0	$65 000
1	$52 000
2	
3	
4	
5	

Example: Detroit City

Detroit is one of the only cities in North America to experience population decline. In the year 1999, the population of Detroit was 950,000 people. The population decreases by 1.5% each year.

1. What number could you multiply the population by to determine the amount of people in the year 2000? (Hint - if 1.5% of the population is lost, how much is remaining? 100% - 1.5%)

2. Use your answer to question #1 to complete the following table.

Year	Population
1999	950, 000
2000	935 750
2001	
2002	
2003	
2004	
2005	

Once you have worked through the above examples, I suggest you watch the Exponential Growth & Decay YouTube video from Mr. Elliott. He will quickly go through the examples above and the rest of the handout with you.

Exploring Exponential Growth & Decay: Summary

Snapchat Party: 1 initial invitation, number of invites triples each day

Contagious Disease: 3 initial infections, number of infections doubles each week

Grocery Store Job: $15.25 per hour, increase of 4% each year

Bald Eages: 3000 nests, increase of 6.8% each year

Car Depreciation: $65 000 car, decrease of 20% value each year

Detroit City: 950,000 people decrease of 1.5% each year

The above are examples of exponential growth or exponential decay.

There is a general formula for exponential growth/decay:

$$y = (a)b^x$$

Where a is always the initial population/amount.

Where b is always the growth or decay rate.

***y* will then give you the population/amount at time period *x*.**

What would the graphs of the functions above look like?

Examples

Rabbits. The population of rabbits doubles every year in Mr. Elliott's back yard. This year Mr. Elliott counts 20 rabbits. How many rabbits will there be in

a) 3 years?

b) 10 years?

Student population. A school board has 58 500 students in the year 2020. They expect to maintain a growth rate of 2.1% per year.

a) Find an equation to model the student population of the school board.

b) Estimate the student population in the year 2025.

c) Estimate the student population in the year 2014.

d) Suppose a different school board has a student population in the year 2020 of 63 200 students. However at this school board the student population is expected to decrease by 2.1% per year. Estimate the student population in 2025.

Cold Virus. Suppose that 250 people in a city are infected with a cold virus. Each one of those people infects on average 3 other people over a course of one week.

a) If this continues, calculate the number of new infections in the 8^{th} week.

b) Suppose instead of infecting an average of 3 other people per week, the average is lowered to 0.8 people. Now calculate the number of new infections in the 8^{th} week.

The population of Calgary is growing exponentially. The population is summarized in the table below:

Year	Population
2005	895,000
2006	949,781
2007	1,007,915

Define an equation that models the population of Calgary and predict the population of Calgary in the year 2015.