MCV4U

Vector & Parametric Equations of Lines

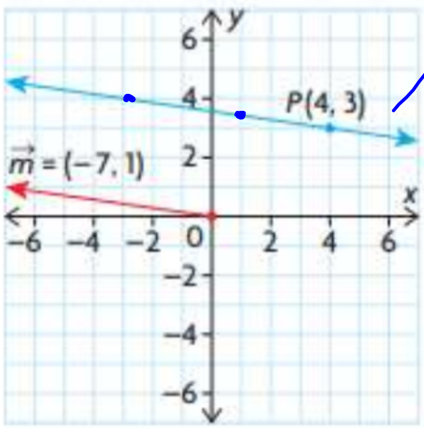
So far you are familiar with finding the equation of a line on the Cartesian plane. You may have found this equation in the form: $y = mx + b$ or (slope y-intercept form) *

The equation of a line can be found if we have the slope (direction of the line) and a point (position of the line).

The same method can be used to develop an equation for a line in $\Re^2$ by finding:

1) A point on the line – expressed as a ***position vector***.
2) A vector in the direction of the line – a ***direction vector.***

$m = -\frac{1}{7}$

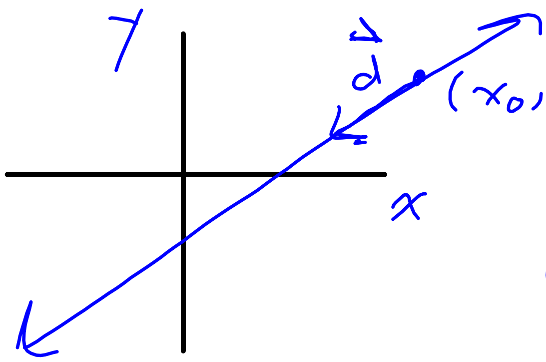


$\boxed{(x, y) = (4, 3) + t(-7, 1)}$

$t \in R$

$(-3, 4)$

Suppose that a line in $\Re^2$ has the point (x_0, y_0) and has the same direction as the vector $\vec{d} = (a, b)$ then the **vector equation** of the line is given by:



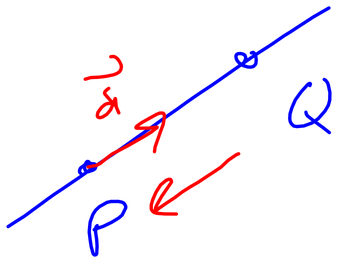
$(x, y) = (x_0, y_0) + t(a, b)$

$\vec{r} = (x_0, y_0) + t(a, b)$

Example: Find the vector equation of the line passing through points P(-1, 1) and Q(5, 5). Then, find the coordinates of 2 other points on the line.

$\overrightarrow{PQ} = (5-(-1), 5-1)$

$\overrightarrow{PQ} = (6, 4)$



$\vec{d} = (3, 2)$

$(x, y) = (-1, 1) + t(3, 2) \quad t \in R$

$t = 1 \quad (x, y) = (-1, 1) + (3, 2) = (2, 3)$

Every value of t specifies on point on the line. Conversely, for every point on the line, there is only one value of t.

$y = 3x - 5$

We can rearrange the vector equation of line to express it as a **parametric equation**:

$$x = -1 + 3t$$
$$y = 1 + 2t$$

$(x_0, y_0) \quad \vec{d} = (a, b)$

$$\begin{cases} x = x_0 + ta \\ y = y_0 + tb \end{cases}$$

Example Find the x and y-intercepts for the line given by the vector equation $\vec{r} = (3,2) + t(1, -2)$.

$x = 3 + t$

$y = 2 - 2t$

x-int $\quad y = 0$

$0 = 2 - 2t$

$2t = 2$

$t = 1$

$x = 3 + 1$

$x = 4$

$(4, 0)$

y-int $\quad 0 = 3 + t$

$t = -3$

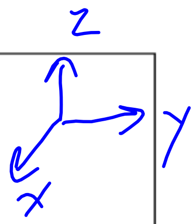
$y = 2 - 2(-3)$

$y = 8$

$(0, 8)$

$y = mx + b$

Using vector and parametric equations allows us to talk about lines in 3-pace (R^3)



The **vector equation** of a straight line in space is given by:

$\vec{r} = (x, y, z) = (x_0, y_0, z_0) + t(a, b, c)$

Where $\vec{d} = (a, b, c)$ is a direction vector for the line (a, b and c are direction numbers)

$t \in R$

(x_0, y_0, z_0) is a the position vector of a *particular point* on the line.

$\vec{r} = (x, y, z)$ is a the position vector for *any point* on the line.

The **parametric equations** of a straight line in space are given by:

$x = x_0 + at$

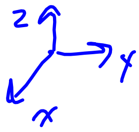
$y = y_0 + bt$

$z = z_0 + ct$

Where (x_0, y_0, z_0) is a particular point on the line

$t \in R$

a,b and c are the direction numbers for the line.



Example: Find vector, parametric and ~~symmetric~~ equations of the line that passes through points A(1, -3, 5) and B(9, 1, 5). Find another point on this line as well.

$\vec{AB} = (8, 4, 0)$ let $\vec{d} = (2, 1, 0)$

$(x, y, z) = (1, -3, 5) + t(2, 1, 0)$ $t \in R$.

$x = 1 + 2t$, $y = -3 + t$, $z = 5$

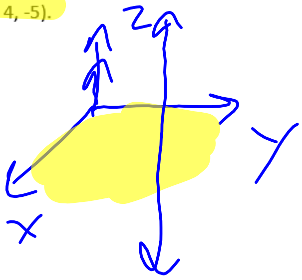
$t = 2$

$x = 1 + 2(2)$ $\quad y = -3 + 2$ $\quad z = 5$

$x = 5$ $\quad y = -1$

$(5, -1, 5)$

Example Find an equation for the line that is perpendicular the xy-plane and going through the point (5, 4, -5).



$\vec{d} = (0,0,1)$ $\hat{k}$

$(x,y,z) = (5,4,-5) + t(0,0,1)$

$x = 5,\ y = 4,\ z = -5 + t$

Example Do the equations below represent the same line?

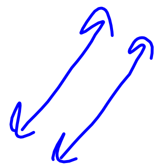
$x = 5 + 2s, y = -4 - 5s, z = -1 + 3s$

$\vec{r} = (-1,11,-4) + t(-4,10,-6)$ $\longrightarrow$ $x = -1 - 4t,\ y = 11 + 10t$

$z = -4 - 6t$

$\vec{d}_1 = (2,-5,3)$

$\vec{d}_2 = (-4,-10,-6)$ $\quad -2\vec{d}_1 = \vec{d}_2$



$(-1, 11, -4)$

$-1 = 5 + 2s$

$-6 = 2s$

$\boxed{s = -3}$

$y = -4 - 5s$

$y = -4 - 5(-3)$

$y = 11$

on line

$(-1, 11, 8)$

$z = -1 + 3s$

$z = -1 + 3(3)$

$z = 8$

not the same line!

Online text Section 8.1 (page 433) #1, 2, 3ab, 4

Section 8.2 (page 449) #1ace, 3, 10ab