Symmetric and Scalar Equations of Lines

Symmetric Equation of a Line

Rearrange the parametric equation of a line to solve for t. The resulting equation is called a symmetric equation of a line.

Example Find the vector, parametric and symmetric equations for the line through the points P(4, -1, 3) and Q(12, -5, 1).

$$\overrightarrow{PQ} = (8, -4, -2) \quad \vec{d} = (4, -2, -1)$$

$$x = 4 + 4t \rightarrow x - 4 = 4t \rightarrow t = \frac{x-4}{4}$$

$$y = -1 - 2t \rightarrow y + 1 = -2t \rightarrow t = \frac{y+1}{-2}$$

$$z = 3 - t \rightarrow t = \frac{z-3}{-1}$$

$$\frac{x-4}{4} = \frac{y+1}{-2} = \frac{z-3}{-1}$$

$$(x_0, y_0, z_0) \quad \vec{d} = (a, b, c) \quad \frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

Example ~~Find the symmetric equation for the line with vector equation~~ $(x, y, z) = (1,2,3) + t(1,0,-2)$

$$\boxed{x - 1 = \frac{z-3}{-2}, \; y = 2}$$

$$x = 1 + t$$

$$z = 3 - 2t$$

$$y = 2$$

Example – Find the vector equation of the line with symmetric equation:

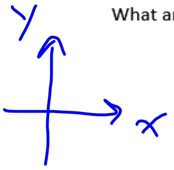
$$\frac{x-1}{3} = \frac{y+5}{1} = \frac{z+3}{-4}$$

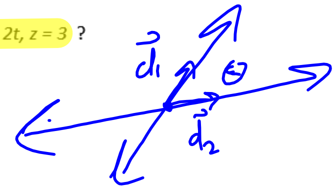
$\vec{d} = (3, 1, -4)$ $(1, -5, -3)$

$(x, y, z) = (1, -5, -3) + t(3, 1, -4)$

$t \in R$

What angle would the line above make with the line: $x = 1 + 4t, z = 1 - 2t, z = 3$?





$\vec{d}_1 = (3, 1, -4)$

$\vec{d}_2 = (4, -2, 0)$

$\vec{d}_1 \cdot \vec{d}_2 = |\vec{d}_1| \, |\vec{d}_2| \cos\theta$

$$\cos\theta = \frac{\vec{d}_1 \cdot \vec{d}_2}{|\vec{d}_1| \, |\vec{d}_2|}$$

$$\cos\theta = \frac{(3,1,-4)\cdot(4,-2,0)}{\sqrt{3^2+1^2+(-4)^2}\,\sqrt{4^2+(-2)^2}}$$

$$\cos\theta = \frac{12 + (1)(-2) + (-4)(0)}{\sqrt{26}\,\sqrt{20}}$$

$$\cos\theta = \frac{10}{\sqrt{26}\,\sqrt{20}}$$

$\theta = 64°$

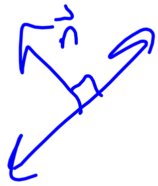
The Normal of a Line

To define the scalar equation of a line we must first define a normal to a line. Any vector that is perpendicular to a line is a called a **normal vector,** or often simply a **normal** to the line.

Find the normal to the line in the example above.

Find two normals to the line given by $\vec{r} = (1,5) + t(3,5)$

$\vec{d} = (3,5)$



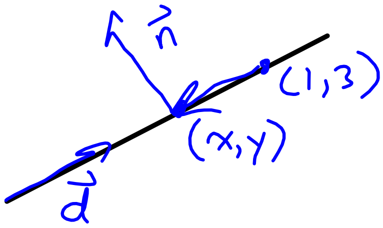
$\vec{n} \cdot \vec{d} = 0$ $\quad n_1 = (-5,3)$ $\quad \vec{n}_2 = (5,-3)$

In general we would tend choose normals that have integer components as small as possible.

Scalar (or Cartesian) Equation

Consider a line with a direction vector of $\vec{d} = (-2,5)$ and point (1,3). We can use the normal to define a new equation (called the scalar or Cartesian equation).

$\vec{n} = (5,2)$



$(x-1, y-3) \cdot (5,2) = 0$

$5x - 5 + 2y - 6 = 0$

$5x + 2y - 11 = 0$

In general a scalar or Cartesian equation of a straight line in 2-space has the form:

Ax + By + C = 0

Example: Find the scalar equation of a line with normal (-3,2) that passes through the point (-3, -7)

$-3x + 2y + c = 0$

$-3(-3) + 2(-7) + c = 0$

$9 - 14 + c = 0$

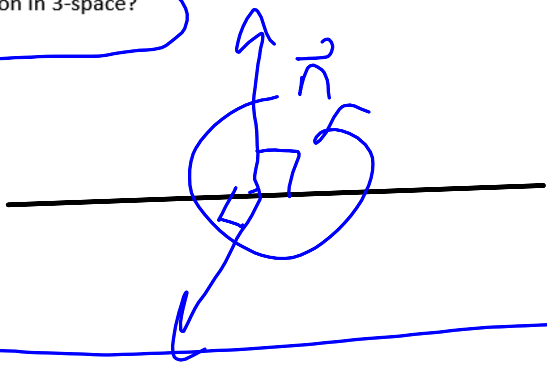
$c = 5$

$-3x + 2y + 5 = 0$ $\quad \vec{n} = (-3,2)$

$\boxed{3x - 2y - 5 = 0}$ $\quad \vec{n} = (3,-2)$

$3x+2y-12=0$

How about a scalar equation in 3-space?



$$\boxed{3x-2y+z-5=0}$$

Assigned Work

Text section 8.3 (page 449) #1bd, 5, 6, 9, 10c
Text section 8.2 (page 443) #7, 8, 9b, 10a