Symmetric and Scalar Equations of Lines

Symmetric Equation of a Line

Rearrange the parametric equation of a line to solve for *t.* The resulting equation is called a symmetric equation of a line.

Example Find the vector, parametric and symmetric equations for the line through the points P(4, -1, 3) and Q(12, -5, 1).

Example Find the symmetric equation for the line with vector equation $(x, y, z) = (1,2,3) + t(1,0,-2)$

Example – Find the vector equation of the line with symmetric equation:

$$\frac{\boldsymbol{x-1}}{\boldsymbol{3}} = \boldsymbol{y+5} = \frac{\boldsymbol{z+3}}{\boldsymbol{-4}}$$

What angle would the line above make with the line: $x = 1 + 4t, z = 1 - 2t, z = 3$?

The Normal of a Line

To define the scalar equation of a line we must first define a **normal** to a line. Any vector that is perpendicular to a line is a called a **normal vector,** or often simply a **normal** to the line.

Find the normal to the line in the example above.

Find two normals to the line given by $\vec{r} = (1,5) + t(3,5)$

In general we would tend choose normals that have integer components as small as possible.

Scalar (or Cartesian) Equation

Consider a line with a direction vector of $\vec{d} = (-2,5)$ and point (1,3). We can use the normal to define a new equation (called the scalar or Cartesian equation).

In general a scalar or Cartesian equation of a straight line in 2-space has the form:

Ax + By + C = 0

Example: Find the scalar equation of a line with normal (-3,2) that passes through the point (-3, -7)

How about a scalar equation in 3-space?

Assigned Work

Text section 8.3 (page 449) #1bd, 5, 6, 9, 10c
Text section 8.2 (page 443) #7, 8, 9b, 10a