MCV4U

Vector & Parametric Equations of Lines

So far you are familiar with finding the equation of a line on the Cartesian plane. You may have found this equation in the form: $y = mx + b$ or (slope y-intercept form)

The equation of a line can be found if we have the slope (direction of the line) and a point (position of the line).

The same method can be used to develop an equation for a line in $\Re^2$ by finding:

1) A point on the line – expressed as a ***position vector***.
2) A vector in the direction of the line – a ***direction vector.***

Suppose that a line in $\Re^2$ has the point (x_0, y_0) and has the same direction as the vector $\vec{d} = (a, b)$ then the **vector equation** of the line is given by:

Example: Find the vector equation of the line passing through points P(-1, 1) and Q(5, 5). Then, find the coordinates of 2 other points on the line.

Every value of *t* specifies on point on the line. Conversely, for every point on the line, there is only one value of *t.*

We can rearrange the vector equation of line to express it as a **parametric equation**:

Example Find the x and y-intercepts for the line given by the vector equation $\vec{r} = (3,2) + t(1,-2)$.

Using vector and parametric equations allows us to talk about lines in 3-pace (R^3)

The **vector equation** of a straight line in space is given by:

$\vec{r} = (x, y, z) = (x_0, y_0, z_0) + t(a, b, c)$

Where $\vec{d} = (a, b, c)$ is a direction vector for the line (a, b and c are direction numbers)

$t \in R$

(x_0, y_0, z_0) is a the position vector of a *particular point* on the line.

$\vec{r} = (x, y, z)$ is a the position vector for *any point* on the line.

The **parametric equations** of a straight line in space are given by:

$x = x_0 + at$

$y = y_0 + bt$

$z = z_0 + ct$

Where (x_0, y_0, z_0) is a particular point on the line

$t \in R$

a,b and c are the direction numbers for the line.

Example: Find vector, parametric and symmetric equations of the line that passes through points A(1, -3, 5) and B(9, 1, 5). Find another point on this line as well.

Example Find an equation for the line that is perpendicular the xy-plane and going through the point (5, 4, -5).

Example Do the equations below represent the same line?

$x = 5 + 2s, y = -4 - 5s, z = -1 + 3s$

$\vec{r} = (-1,11,-4) + t(-4,10,-6)$

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