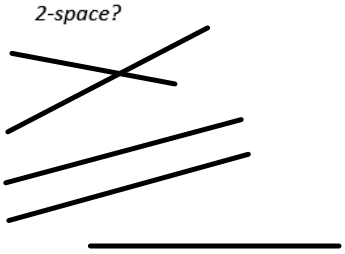
May 15, 2020

MCV4U

Intersection of 2 Lines

What are the possibilities for the intersection of 2 lines in:

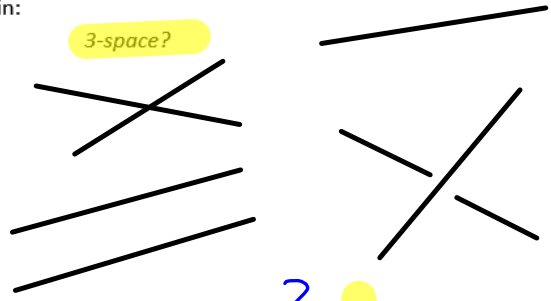
2-space?



Linear Equations 2

Variables: 2

3-space?



Linear Equations 3

Variables: 2

Example (2-space): Find the intersection of the lines: $2x - y + 8 = 0$ (1) and $5x + 3y - 13 = 0$ (2)

$y = 2x + 8$ sub into (2) $5x + 3(2x+8) - 13 = 0$

$5x + 6x + 24 - 13 = 0$

$11x + 11 = 0$

$x = -1$

if $x = -1$,

$y = 2(-1) + 8$

$y = 6$

$(-1, 6)$

algebraic

Example (3-space): Find the intersection of the lines: $\vec{r}_1 = (-1,1,0) + t(3,4,-2)$ and $\vec{r}_2 = (-1,0,-7) + s(2,3,1)$.

Solution:

Start with the parametric equations:

$x = -1 + 3t$
$y = 1 + 4t$
$z = -2t$

and

$x = -1 + 2s$
$y = 3s$
$z = -7 + s$

① $-1+3t = -1+2s \rightarrow -2s+3t = 0$
② $1+4t = 3s \rightarrow -3s+4t = -1$
③ $-2t = -7+s \rightarrow -s-2t = -7$

Combine to get:

$+$ ② $-3s+4t = -1$
$2\times$ ③ $-2s-4t = -14$

$-5s = -15$
$s = 3$

$-2(3) - 4t = -14$
$-6 - 4t = -14$
$t = 2$

check in ① $RS = 0$
$LS = -2(3) + 3(2)$
$LS = 0$

✓ $LS = RS$
$s = 3$ $t = 2$ ☺

$s = 3$

$x = -1+2s$
$y = 3s$
$z = -7+s$

$x = -1+2(3)$
$x = 5$
$y = 3(3)$
$y = 9$

$z = -7+3$
$z = -4$

$(5, 9, -4)$

Example: Find the intersection of the lines: $\vec{r}_1 = (2,1,0) + t(1,-1,1)$ and $\frac{x-3}{2} = \frac{y}{3} = z + 1$

$\vec{d}_1 = (1,-1,1)$ $\vec{d}_2 = (2,3,1)$

Solution:

Start with the parametric equations:

$x = 2 + t$
$y = 1 - t$
$z = t$

and

$x = 3 + 2s$
$y = 3s$
$z = s - 1$

(1) $2 + t = 3 + 2s \longrightarrow -2s + t = 1$

(2) $1 - t = 3s \longrightarrow -3s - t = -1$

(3) $t = s - 1 \longrightarrow -s + t = -1$

(2) $-3s - t = -1$

(3) $-s + t = -1$

$-4s = -2$

$s = \frac{1}{2}$

$-\frac{1}{2} + t = -1$

$t = -1 + \frac{1}{2}$

$t = -\frac{1}{2}$

check in (1)

RS $= 1$

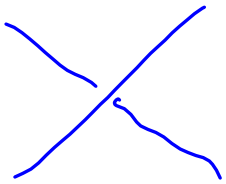
LS $= -2(\frac{1}{2}) + \frac{1}{2}$

$= -1 + \frac{1}{2}$

$= -\frac{1}{2}$

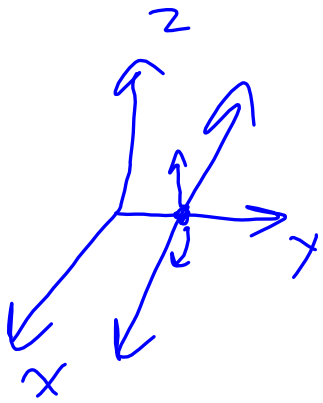
LS $\neq$ RS

NO solution



skew lines

Example: Find the y-intercept for the line $\vec{r}_1 = (2,1,-1) + t(1,-1,1)$



Solution:

Start with the parametric equations:

$x = 2 + t$

$y = 1 - t$

$z = -1 + t$

$x = 0$ and $z = 0$

$0 = 2 + t$ $\quad$ $0 = -1 + t$

$t = -2$ $\quad$ $t = 1$

So is no y-intercept

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