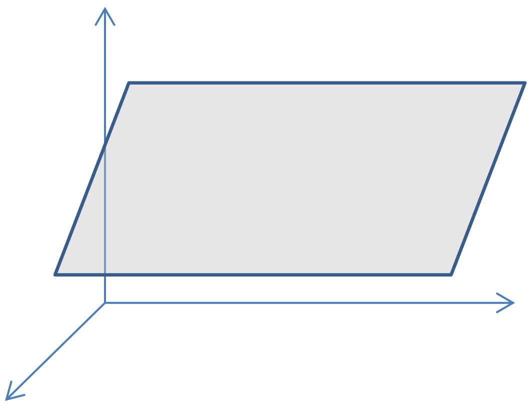
MCV4U

Equations of Planes

Planes are flat 2-D surfaces. The Cartesian (xy-plane) is an example of a plane. We often draw planes as parallelograms on a flat surface, even though the planes extend in all directions.

Using a vector equation of a line, we define a line by establishing position on a line (initial point) then adding a scalar multiple of a direction vector to arrive at new points. We can define a plane using a vector equation similar to a line.



The **vector equation** of a plane is given by:

$(x, y, z) = (x_0, y_0, z_0) + t(a_1, b_1, c_1) + s(a_2, b_2, c_2)$ where $s, t \in R$.

We then can define **parametric equations** of plane of the form:

$x = x_0 + sa_1 + ta_2$

$y = y_0 + sb_1 + tb_2$

$z = z_0 + sc_1 + tc_2$

where $s, t \in R$.

Any three non-collinear points define a plane.

Find the vector and parametric equations of the plane formed by the points A(1, 2, 3), B(4, 5, -3) and C(0,0,6).

Does the point (-7, 3, -1) lie in the plane formed above?

Find the equation of a plane that containing two intersecting lines: $\vec{r}_1 = (4,7,3) + t(1,4,3)$ and $\vec{r}_2 = (-1,-4,6) + s(-1,-1,3)$

Find the equation of the plane containing the lines $\frac{x-3}{2} = \frac{y}{-3} = \frac{z+4}{3}$ and $x = 2t, y = -3t + 5, \quad z = 3t - 1.$

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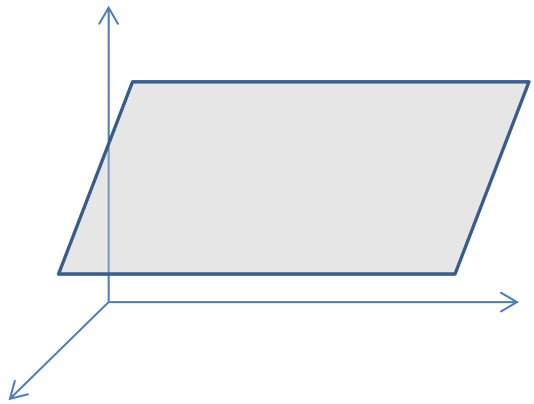
Instead of using 2 direction vectors to "establish a direction for our plane" we could use the **normal to the plane.**

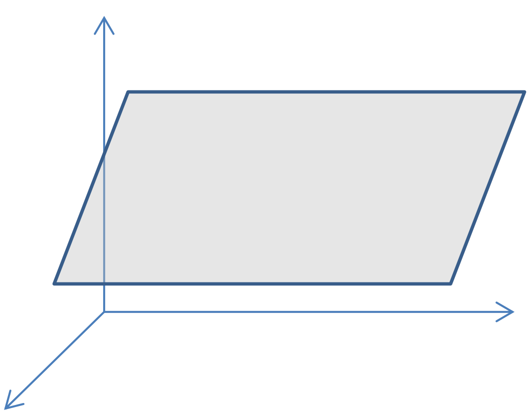


Example Find a normal to the plane $\vec{r} = (1,2,9) + t(-1,0,1) + s(1,-3,4)$.

Is the normal unique?

The normal to a plane will be perpendicular to any vector in the plane. We can use this fact to derive the scalar equation to a plane.

Let P(x,y,z) be any point in the plane with normal (A,B,C). Let Q (x_0,y_0,z_0) be a particular point (given point) in the plane.



What is the scalar equation for the plane from the example on the previous page?

The scalar equation is unique.

Example: Find the scalar equation of the plane containing the 3 points A(1,1,1), B(2, 0, -1) and C(0, -1, 4).

Example: Find the scalar equation of the plane containing the point (4, 3, -2) and is parallel to the xz-plane.

Text page 468 #6a, 7, 8, 9a, 11 and page 480 #5c, 7, 9