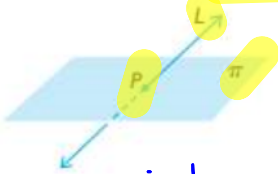
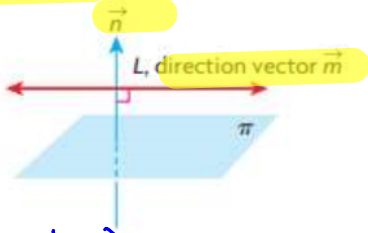
Intersection of a Line and a Plane

Start by considering the geometric possibilities for the intersection of a line and a plane.



one point



$\vec{m}\cdot\vec{n}=0$

no solution

infinite solutions (line)

$\vec{m}\cdot\vec{n}=0$

$$\boxed{\begin{array}{l}3x=3x+1\\ 3x+1=3x+1\end{array}}$$

one equation, one unknown.

Example 1 line: $\frac{x-1}{2}=\frac{y+6}{3}=\frac{z+5}{2}$ and plane: $4x-2y+z-19=0$.

$\vec{d}=(2,3,2)$ $\quad\vec{n}=(4,-2,1)$

$$\vec{d}\cdot\vec{n}=4\neq 0$$

$x=1+2t,\ y=-6+3t,\ z=-5+2t$

sub into $4x-2y+z-19=0$

$$4(1+2t)-2(-6+3t)-5+2t-19=0$$

$$4+8t+12-6t-5+2t-19=0$$

$$4t-8=0$$

$$t=2$$

$x=1+2(2)$ $\quad y=-6+3(2)$ $\quad z=-5+2(2)$

$x=5$ $\quad y=0$ $\quad z=-1$

$$(5,0,-1)$$

Example 2

line: $\vec{r} = (0,1,-4) + (2,-1,1)t$ and plane: x + 4y + 2z – 4 = 0

$x = 2t,\ y = 1 - t,\ z = -4 + t$

$2t + 4(1-t) + 2(-4+t) - 4 = 0$

$2t + 4 - 4t - 8 + 2t - 4 = 0$

$0t - 8 = 0$

$-8 = 0$ no solution

$\vec{d} = (2,-1,1) \quad \vec{n} = (1,4,2)$

$\vec{d} \cdot \vec{n} = 0 \quad \vec{d} \perp \vec{n}$

Example 3

line: $(3+14s,-2-5s,1-3s)$ and plane: $x+y+3z-4=0$

(x,y,z)

$3+14s-2-5s+3(1-3s)-4=0$

$3+14s-2-5s+3-9s-4=0$

$0s-0=0$

$0=0$

intersection is the line.

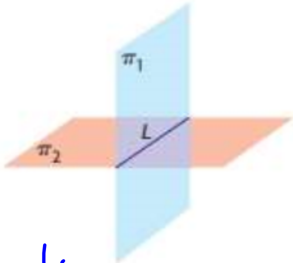
$\vec{d}=(14,-5,-3)$ $\vec{n}=(1,1,3)$

$\vec{d}\cdot\vec{n}=0$

Intersection of Two Planes

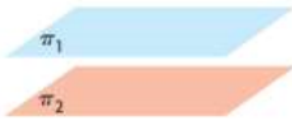
Start by considering the geometric possibilities for the intersection of 2 planes. Include specific properties of each plane that would help you determine the scenario.

Case 1: Two Planes Intersecting along a Line



line

Case 2: Two Parallel Planes



no intersection

Case 3: Two Coincident Planes

plane

2 equations
3 unknowns

Example 1

Find the point of intersection of the planes 2x - 2y + 5z + 10 = 0 and 2x + y –4 z + 7 = 0.

$$(1)\quad 2x - 2y + 5z + 10 = 0$$
$$-\quad (2)\quad 2x + y - 4z + 7 = 0$$

$$(1)-(2)\quad -3y + 9z + 3 = 0$$
$$-3y = -9z - 3$$
$$y = 3z + 1 \quad *$$

sub. into (2)

$$2x + 3z + 1 - 4z + 7 = 0$$
$$2x - z + 8 = 0$$
$$2x = z - 8$$
$$x = \tfrac{1}{2}z - 4 \quad *$$

Let $z = t$

$$y = 3t + 1$$
$$x = \tfrac{1}{2}t - 4$$
$$t \in R$$

$\xrightarrow{\text{better}}$

$$\boxed{\begin{aligned} x &= t - 4 \\ y &= 6t + 1 \\ z &= 2t \\ t &\in R \end{aligned}}$$

Example 2

Find the intersection between the planes x + y + z – 1 = 0 and 2x – 3y –z + 2 = 0

$$\text{①}\quad x + y + z - 1 = 0$$
$$+\ \text{②}\quad 2x - 3y - z + 2 = 0$$

$$3x - 2y + 1 = 0$$
$$3x = 2y - 1$$
$$x = \tfrac{2}{3}y - \tfrac{1}{3}$$

sub into ①

$$\tfrac{2}{3}y - \tfrac{1}{3} + y + z - 1 = 0$$
$$2y - 1 + 3y + 3z - 3 = 0$$
$$z = -\tfrac{5}{3}y + \tfrac{4}{3}$$

$$x = \tfrac{2}{3}t - \tfrac{1}{3}$$
$$y = t$$
$$z = -\tfrac{5}{3}y + \tfrac{4}{3}$$
$$t \in R$$

$$\boxed{\begin{aligned} x &= 2t - \tfrac{1}{3} \\ y &= 3t \\ z &= -5t + \tfrac{4}{3} \\ t &\in R \end{aligned}}$$

Example 3

Solve the system of equations below:

(1) x + 4y – 3z + 6 = 0
(2) 2x + 8y - 6z – 9 = 0

$\vec{n}_1 = (1, 4, -3)$

$\vec{n}_2 = (2, 8, -6)$

$2\vec{n}_1 = \vec{n}_2$

$\vec{n}_1 \parallel \vec{n}_2$

no solution

2(1) − (2) $\quad -3 = 0$

$-3 \neq 0$

Example 4

Find the point of intersection between the planes $3x - 5y - z + 2 = 0$ and $9x - 15y - 3z + 6 = 0$

① $3x - 5y - z + 2 = 0$
② $9x - 15y - 3z + 6 = 0$

$\vec{n}_1 = (3, -5, -1)$
$\vec{n}_2 = (9, -15, -3)$

intersection is plane

$3① - ②$ $\quad 0 = 0$

$3x - 5y - z + 2 = 0$

Text page 497 #4a,5a,7 and page 517 #6,7,8,9