Introduction to Annuities

Warmup Problem

Suppose Ben has $5000 in his savings account. It collects interest at 2.5%/a compounded monthly. How much will Ben have in exactly one year towards the purchase of a vehicle?

$$A = P(1+i)^n$$
$$A = 5000\left(1 + \frac{.025}{12}\right)^{12}$$
$$= \$5126.44$$

Consider this situation:

Ben wants to save money to purchase a new vehicle. At the end of every month, he deposits $450 into a savings account that collects interest at 2.5%/a compounded monthly. How much will Ben have in his account after 12 months?

An annuity is a series of payments made at regular intervals. (For a simple annuity the compounding periods and payment periods are equal).

Ben's Savings: Timeline Solution

$A = P(1+i)^n$

Month	Deposit	
1	$450	$\rightarrow 450\left(1+\frac{.025}{12}\right)^{11} \rightarrow \$460.42$
2	$450	$\rightarrow 450\left(1+\frac{.025}{12}\right)^{10} \rightarrow \$459.46$
3	$450	$\rightarrow 450\left(1+\frac{.025}{12}\right)^{9}$
4	$450	$\rightarrow 450\left(1+\frac{.025}{12}\right)^{8}$
5	$450	$\rightarrow 450\left(1+\frac{.025}{12}\right)^{7}$
6	$450	$\rightarrow 450\left(1+\frac{.025}{12}\right)^{6}$
7	$450	$\rightarrow 450\left(1+\frac{.025}{12}\right)^{5}$
8	$450	$\rightarrow 450\left(1+\frac{.025}{12}\right)^{4}$
9	$450	$\rightarrow 450\left(1+\frac{.025}{12}\right)^{3}$
10	$450	$\rightarrow 450\left(1+\frac{.025}{12}\right)^{2}$
11	$450	$\rightarrow 450\left(1+\frac{.025}{12}\right)^{1}$
12	$450	$\rightarrow \$450$

Final Value of Ben's Savings: $\$5462.31$

Thankfully, there is a formula that can be used to determine the final value of a simple annuity such as the one above.

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

or $FV i = R[(1+i)^n - 1]$

$$R = \frac{FV i}{[(1+i)^n - 1]}$$

FV is the future value of the annuity
R is the regular payment
i is the interest rate (per compounding period)
n is the number of payments.

To calculate the final value of Ben's annuity we could use this formula with:

$$FV = \frac{450\left[\left(1+\frac{.025}{12}\right)^{12} - 1\right]}{\left(\frac{.025}{12}\right)}$$

Example

In order to save for her own college education, Rachel's grandparents provide her with a gift of $1000 on her birthday. Rachel invests this money at 2.1%/a compounded annually. How much will she have after 18 years?

every

$$FV = R[(1+i)^n - 1]$$

$$= \frac{1000[(1+.021)^{18} - 1]}{.021}$$

$$= \$21602.94$$

Example

Gary is 35 years old and starts saving for retirement. He is paid biweekly and $100 from each paycheque is deposited into an RRSP that pays 3%/a compounded bi-weekly. How much money will he have if he wants to retire at age 60? How much interest would he earn?

30%

Bi-weekly = every other week

RRSP = registered retirement savings plan

26

$n = 25 \times 26$

$n = 650$

$i = \frac{.03}{26}$

$FV = \frac{R[(1+i)^n - 1]}{i}$

$= \frac{100\left[\left(1+\frac{.03}{26}\right)^{650} - 1\right]}{\left(\frac{.03}{26}\right)}$

$= \$96727.36$

interest

$650 \times 100 = \$65000$

interest is

$\$96727.36 - \$65000 = \$31727.36$

Example

Kari wishes to backpack around Europe in 3 years. She has decided that she needs $5000 in savings to do this. She has decided to contribute to a savings account every month. The savings account pays interest at 1.75%/a compounded monthly. How much does she need to deposit in the savings account every month?

$FV = 5000$

$R = ?$

$n = 3 \times 12$

$= 36$

$R = \frac{FVi}{[(1+i)^n - 1]}$

$R = \frac{5000\left(\frac{.0175}{12}\right)}{\left[\left(1+\frac{.0175}{12}\right)^{36} - 1\right]}$

$R = \$135.38$

TVM Solver

In regular life, most individuals do not use this formula. One option is to search and use a TVM solver (Time Value of Money)

Use a **TVM Solver** to answer the following:

1) For 4 years you put $20 a week into a savings account that pays 3.25% interest. What would the final value of this annuity be? How much interest would you have earned?

$4 \times 52 = 208$

$FV = \$4441.03$

interest $= 4441.03 - (20 \times 208)$

$= \$281.03$

2) Mr. Elliott wants to have one million dollars saved. He will make monthly payments for the next 20 years to achieve this goal. The money will collect interest at 2%/a compounded monthly. How much does he need to save each month?

$FV = 1000000$ $R = ?$

20×12

$= 240$

$\$3392.17$ each month!