Intersection of 3 Planes – Assigned Problems

Full solutions are available online for questions 1 - 3. Be sure to check your answers.

1. Give a geometrical interpretation of each system of equations below by examining the normal and equations. Describe the intersection (if possible).

a)
$$\begin{aligned} x + 2y - z &= 1 \\ x - 5y + 4z &= -4 \\ 3x - y + 2z &= 2 \end{aligned}$$

b)
$$\begin{aligned} x + 2y + z &= 12 \\ 2x - y + z &= 5 \\ 3x + y - 2z &= 1 \end{aligned}$$

c)
$$\begin{aligned} x - 3y &= -7 \\ 2x + y - z &= 3 \\ 4x + 2y - 2z &= 8 \end{aligned}$$

d)
$$\begin{aligned} -2x + 4y + 6z &= -2 \\ 4x - 8y - 12z &= 4 \\ x - 2y - 3z &= 1 \end{aligned}$$

2. Solve each system of equations below. Then, give a geometric interpretation of each system by describing the orientation of the planes in space (or just provide a sketch).

a.
$$\begin{aligned} x + 2y + z &= 12 \\ 2x - y + z &= 5 \\ 3x + y - 2z &= 1 \end{aligned}$$

b.
$$\begin{aligned} x - y + 2z &= 4 \\ 2x - 2y + 4z &= 7 \\ 3x - 3y + 6z &= 11 \end{aligned}$$

c.
$$\begin{aligned} x + y - z &= 5 \\ 2x + 2y - 4z &= 6 \\ x + y - 2z &= 3 \end{aligned}$$

d.
$$\begin{aligned} -2x + 4y + 6z &= -2 \\ 4x - 8y - 12z &= 4 \\ x - 2y - 3z &= 1 \end{aligned}$$

e.
$$\begin{aligned} x + y + 2z &= 2 \\ x - y - 2z &= 5 \\ 3x + 3y + 6z &= 5 \end{aligned}$$

f.
$$\begin{aligned} x + 3y + 5z &= 10 \\ 2x + 6y + 10z &= 18 \\ x + 3y + 5z &= 9 \end{aligned}$$

g.
$$\begin{aligned} x - 3y - 2z &= 9 \\ x + 11y + 5z &= -5 \\ 2x + 8y + 3z &= 4 \end{aligned}$$

h.
$$\begin{aligned} x + y + 2z &= 6 \\ x - y - 4z &= -2 \\ 3x + 5y + 12z &= 27 \end{aligned}$$

i.
$$\begin{aligned} 2x + y + z &= 0 \\ x - 2y - 3z &= 0 \\ 3x + 2y + 4z &= 0 \end{aligned}$$

3. For what value of k will the following set of planes intersect in a line?

$$\begin{aligned} x - 2y - z &= 0 \\ x + 9y - 5z &= 0 \\ kx - y + z &= 0 \end{aligned}$$

4. If you would like some extra practice, try the green (online) textbook page 533 #13.