

Loans as an Annuity

$$PV \rightarrow R$$

Warm-up

Mr. Elliott wins 1 million dollars in the lottery. He quits teaching. He puts the money into an annuity that pays 6%/a compounded monthly. He will make equal monthly withdrawals for the next 50 years. How much will each withdrawal be worth?

$$n = 50 \times 12 = 600$$

$$PV = \frac{R [1 - (1+i)^{-n}]}{i}$$

$$\text{or } R = \frac{PVi}{[1 - (1+i)^{-n}]}$$

$$R = \frac{1000000 \left(\frac{.06}{12}\right)}{\left[1 - \left(1 + \frac{.06}{12}\right)^{-600}\right]}$$

$$R = \$5264.05$$

LOAN – an agreement by a person or business to borrow money from the bank and pay it back with interest.

Suppose Sammy borrows \$25,000 to purchase a new vehicle. He will be 7%/a compounded monthly and make monthly payments for the next 5 years to pay off the loan. How much is each payment?

$$R = \frac{25000 \left(\frac{.07}{12}\right)}{\left[1 - \left(1 + \frac{.07}{12}\right)^{-60}\right]}$$

$$n = 5 \times 12 = 60$$

$$R = \$495.03$$



Erika will also borrow money at 7%/a compounded monthly. The loan is amortized over 5 years. If she can only afford \$250 monthly payments, how much can she afford to borrow?

$$n = 60$$

$$PV = \frac{R [1 - (1+i)^{-n}]}{i}$$

$$\$12625.50$$

How much interest did Erika pay in total?

$$\begin{aligned} & \$250 \times 60 \\ = & \$15000 \end{aligned}$$

$$\begin{aligned} & \$15000 - \$12625.50 \\ = & \$2374.50 \end{aligned}$$

A mortgage is a special type of loan used to purchase a house. Find the monthly payments on a mortgage for \$280,000 amortized over 25 years at an interest rate of 3%/a. (compounded monthly).

$$R = \frac{PVi}{[1 - (1+i)^{-n}]}$$

$$i = \frac{.03}{12}$$

$$n = 12 \times 25 \\ = 300$$



$$R = \$1327.79$$

When purchasing a home a person usually saves up money to make a **down payment**. Suppose you want to purchase a home in 5 years. You save \$150 per month to go towards the down payment. The money collects interest at 2.5%/a compounded monthly. How much money will you have saved for a down payment?

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

$$= \frac{150 \left[\left(1 + \frac{.025}{12}\right)^{60} - 1 \right]}{\left(\frac{.025}{12}\right)}$$

$$= \$9576.08$$

Types of Loans

Credit Card – If you do not pay your credit card bill, it is transferred into a loan. The interest rate is often very high on a credit card (anywhere from 15% to as much as 30%)

Short Term Loan – A loan over a short period of time (1 – 6 years). Interest rates start at around 5% - 8%.

Auto Loan – A loan used to purchase a vehicle. Interest rates usually around 5 – 8%, but might be lower for a brand new vehicle. In this case, your vehicle is collateral for the loan.

Line of Credit – This type of loan is a bank account that has a negative balance. You borrow money as you need it. Very common for students, or people who need money in an emergency. Similar interest rate to a short-term loan.

OSAP – (Ontario Student Assistance Plan) – A special type of student loan offered by the government of Ontario. While you are a student there is no interest on the loan. You also do not need to start paying off the loan until 6 months after graduation from college/university. Interest rate is similar to a short-term loan.

Mortgage – A loan used to purchase a home. Mortgages are typically 15 – 25 years in length. (**Amortization period** is 15-25 years). Interest rates are much lower than other loan (as low as 2%). A down payment (at least 5% in Canada) is required to obtain the mortgage.

Interest rates above are just simple guidelines..

What affects the interest rate on a loan?

- the type of loan
- your credit history
- your income
- prime interest rates (which go up and down as the Bank of Canada)
- the financial institution (you can “shop” around).

We will learn more about Automotive Loans and Mortgages next week!