Present Value of Annuities

$450 \times 48 = 21{,}600$

Warm-up

An ambitious grade 9 student decides to start saving for their University education. At the end of every month the student deposits $450 into a savings account that pays 3.5%/a compounded monthly. After exactly 4 years, how much money would the student have saved?

$n = 12 \times 4$
$= 48$

$$FV = \frac{R[(1+i)^n - 1]}{i}$$

$$FV = \frac{450\left[\left(1+\frac{.035}{12}\right)^{48} - 1\right]}{\left(\frac{.035}{12}\right)} \rightarrow FV = \$23\,148.94$$

Consider the following type of annuity…

A student wants to have $500 every month for the first year of college to help pay for living expenses. They will keep the money in a savings account (paying 3%/a compounded monthly) and make regular withdrawals at the end of each month. What starting balance (present value) of the account is needed at the beginning of the year in order to have this money available?

$\$500 \times 12$
$= \$6000$

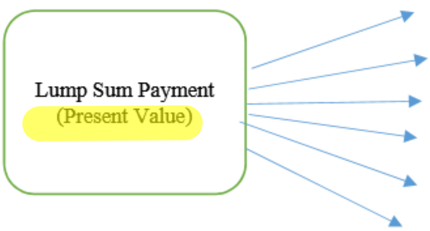
Above is an example of a totally different type of annuity. In this annuity there is a starting balance and a person withdraws money as the balance collects interest. At the end of the annuity the money is all gone. There is a "present value" formula for this type of annuity:

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

$R \rightarrow$ withdrawal (\$)
$i \rightarrow$ interest rate
$n \rightarrow$ number of withdrawals

or $$R = \frac{PV\,i}{[1-(1+i)^{-n}]}$$

Regular Pay Outs Out (Withdrawals) Over Time



$500
$500
$500
⋮
$500

sin30

MCF3M

D.A.L.

$n = 12$

Use the formula to find out how much the student above needs to have saved.

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

$$PV = \frac{500[1-(1+\frac{.03}{12})^{-12}]}{(\frac{.03}{12})}$$

$PV = \$5903.63$

Example

Mr. Elliott wants to live off of $3500 a month when he retires. His retirement income plan will pay 6%/a compounded monthly. If Mr. Elliott needs retirement income for 30 years after he retires, how much must he have saved by the time he retires?

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

$n = 12 \times 30$

$n = 360$

$$PV = \frac{3500[1-(1+\frac{.06}{12})^{-360}]}{(\frac{.06}{12})}$$

360×3500

$= 1.26$ million

$= \$583\ 770.65$

6%
30 years

Example

Suppose "Joe" retires with $450,000. Using the same amounts from the last example, what monthly payment amount could Joe receive from an annuity? How much interest would Joe earn?

$$PV = \frac{R[1-(1+i)^{-n}]}{i}$$

$$PVi = R[1-(1+i)^{-n}]$$

$$\boxed{R = \frac{PVi}{[1-(1+i)^{-n}]}}$$

$$R = \frac{450000\left(\frac{.06}{12}\right)}{\left[1-\left(1+\frac{.06}{12}\right)^{-360}\right]}$$

$$R = \$2\ 697.98$$

Example: Winning the Lottery PV

Suppose you win a five million dollar jackpot in the lottery. You decide to invest the money into an annuity paying 4%/a compounded weekly. What weekly payment could you withdraw that would allow the money to last for the next 75 years? **Use a TVM Solver.**

$$R = \frac{PVi}{[1-(1+i)^{-n}]}$$

$n = 75 \times 52$
$= 3900$

$$\boxed{\$4047.92}$$

$\$4047.92 \times 52 \times 75$

$= \$15\ 786\ 888$

$\$15\ 786\ 888 - \$5\ 000\ 000$

$$\boxed{= \$10\ 786\ 888}$$

interest