Euler's Number and the Natural Logarithms

1. Let $f(x) = (e^x + 1)^2$. Determine $f'(0)$.

2. Find the derivative of each of the following:

   a) $y = x^4 e^x$ b) $y = \frac{e^x}{x}$ c) $y = xe^{-2x}$

   d) $f(x) = \ln x^{-2}$ e) $y = 6\ln(x^2 + 3x)$ f) $y = \sqrt{\ln x}$ g) $y = x^4 \ln x$

3. Find the equation of the tangent to the curve $y = e^{-x}$ at the point where x = -1.

4. Let $y = x^2 e^{-x}$. Find the intervals of increase and decrease of the graph.

5. Find the points on the graph of $y = (3 - x^2)e^x$, where the tangent line is horizontal.

6. Express as a single logarithm:

$-5\ln 2x + 6\ln\text{x}$

7. Find the equation to the tangent to $y = x\ln x$ at:
   a) x = e
   b) that has slope 3.

ANSWERS

1. 4

2. a) $\frac{dy}{dx} = 4x^3e^x + x^4e^x$ b) $\frac{dy}{dx} = \frac{xe^x - e^x}{x^2}$ c) $\frac{dy}{dx} = e^{-2x} - 2xe^{-2x}$ d) f'(x)= -2/x

e) $\frac{dy}{dx} = \frac{6(2x+3)}{x^2+3x}$ f) $\frac{dy}{dx} = \frac{1}{2x\sqrt{\ln x}}$ g) $\frac{dy}{dx} = x^3 + 4x^3\ln x$

3. y = -ex **4.** Increasing 0 < x < 2 and decreasing for x<0 and x > 2.

5. (1, 2e) and $\left(-3, \frac{-6}{e^3}\right)$ **6.** $\ln\left(\frac{x}{32}\right)$ **7.** a) 2x – y – e = 0 b) 3x – y –e^2 = 0