MCV4U          Logarithmic Differentiation

**1.** Differentiate.
   (a) $y = 5^x$       (b) $y = (0.47)^x$       (c) $y = (52)^{2x}$
   (d) $y = 5(2)^x$    (e) $y = 4(e)^x$        (f) $y = -2(10)^{3x}$

**2.** Find $\frac{dy}{dx}$ for each function.
   (a) $y = \log_2 x$        (b) $y = \log_3 x$        (c) $y = 2\log_4 x$
   (d) $y = -3\log_7 x$      (e) $y = -(\log x)$       (f) $y = 3\log_6 x$

**3. Knowledge and Understanding:** Find $\frac{dy}{dx}$ for each function.
   (a) $y = \log_3 (x + 2)$        (b) $y = 4^{2x + 1}$        (c) $y = \log_8 (2x)$
   (d) $y = -3\log_3 (2x + 3)$     (e) $y = 400(2)^{x + 3}$    (f) $y = 8^{2x + 5}$
   (g) $y = \log_{10} (5 - 2x)$    (h) $y = 10^{2 - 3x}$       (i) $y = \log_8 (2x + 6)$
   (j) $y = 100(1.02)^x$

**4.** Paula invests \$1200 at 12%/a, compounded annually.
   (a) What is the amount of the investment after five years?
   (b) Determine the equation for $A(t)$, the amount after $t$ years.
   (c) Determine an equation for the rate of change of the money's value.
   (d) What is the rate of change of the value at five years?

**5.** Differentiate.
   (a) $y = x^5 \times (5)^x$          (b) $y = \log_7 (x^2 + x + 1)$
   (c) $y = 3^x \log_3 x$             (d) $y = \frac{2^{4x}}{x^3}$
   (e) $y = 2x \log_4 x$              (f) $y = 3.2(10)^{0.2x}$
   (g) $y = 2^x \ln x$               (h) $y = x(3x)^{x^2}$
   (i) $y = 3^{\ln x}$               (j) $y = \frac{\log_5 3x^4}{5^{2x}}$

**6.** Jim invests \$5000 at 8%/a, compounded annually.
   (a) Determine the equation for the amount, $A(t)$, after $t$ years.
   (b) Determine an equation for the rate of change of the money's value.
   (c) Determine the rate of change of the value at 10 years.
   (d) Suppose that interest was compounded quarterly rather than annually.
       What would be the rate of change at 10 years?

**7.** Find the equation of the tangent to the curve $y = 3(2^x)$ at the point where
   $x = 3$.

**8.** Find the equation of the tangent to the curve $y = 10^x$ at point $(1, 10)$.

**9. Application:** A certain radioactive material decays exponentially. The
   percent, $P$, of the material left after $t$ years is given by $P(t) = 100(1.2)^{-t}$.
   (a) Determine the half-life of the substance.
   (b) How fast is the substance decaying at the point where the half-life is
       reached?

10. Sketch the graph of $f(x) = \dfrac{2x^2}{e^x}$ by determining all turning points and points of inflection.

Answers:

1. (a) $5^x \ln 5$        (b) $(0.47)^x \ln 0.47$
(c) $(52)^{2x}(\ln 52)(2)$       (d) $5(2^x)(\ln 2)$
(e) $4e^x$             (f) $-6(10^{3x})\ln 10$

2. (a) $\frac{1}{x \ln 2}$    (b) $\frac{1}{x \ln 3}$    (c) $\frac{2}{x \ln 4}$
(d) $\frac{-3}{x \ln 7}$    (e) $\frac{-1}{x \ln 10}$    (f) $\frac{3}{x \ln 6}$

3. (a) $\frac{1}{(x+2)(\ln 3)}$     (b) $(2^{4x+4})(\ln 2)$
(c) $\frac{1}{x \ln 8}$         (d) $\frac{-6}{(2x+3)\ln 3}$
(e) $(3200)(2^x)(\ln 2)$     (f) $(32\,768)(\ln 64)(2^{6x})$
(g) $\frac{-2}{(5-2x)(\ln 10)}$     (h) $-3(10)^{2-3x}(\ln 10)$
(i) $\frac{1}{(x+3)(\ln 8)}$      (j) $100(1.02)^x(\ln 1.02)$

4. (a) $\$ 2114.81$       (b) $1200(1.12)^t$
(c) $1200(1.12)^t(\ln 1.12)$    (d) $\$ 239.67/\text{year}$

5. (a) $x^5(5^x)(\ln 5) + 5x^4 5^x$    (b) $\frac{(2x+1)}{(x^2+x+1)(\ln 7)}$
(c) $\frac{3^x}{x \ln 3} + 3^x \ln x$      (d) $\frac{2^{4x}(4x \ln 2 - 3)}{x^4}$
(e) $(1 + \ln x)\frac{1}{\ln 2}$      (f) $(0.64)(10)^{0.2x}(\ln 10)$
(g) $2^x\left(\frac{1}{x}\right) + 2^x(\ln 2)(\ln x)$    (h) $x(3x)^{x^2}\left(\frac{1}{x} + x + 2x \ln 3x\right)$
(i) $3^{\ln x}(\ln 3)\left(\frac{1}{x}\right)$      (j) $\left[\frac{1}{5^{2x} \ln (5)}\right]\left[\frac{4}{x} - 2 \ln (5) \ln (3x^4)\right]$

6. (a) $5000(1.08)^t$       (b) $5000(1.08)^t(\ln 1.08)$

(c) $\$830.77/\text{year}$      (d) $\$874.50/\text{year}$

7. $(24 \ln 2)x - y + 24 - 72 \ln 2 = 0$
8. $(10 \ln 10)x - y + 10 - 10 \ln 10 = 0\%/\text{year}$
9. (a) $t = 3.802$ years     (b) $-9.12\%/\text{year}$
10.
          solution on course web page.