Unit 7, Lesson 3 solutions

3. d) $y = -3\log_3(2x+3)$

$y = \log_3(2x+3)^{-3}$

$3^y = (2x+3)^{-3}$

$\ln 3^y = \ln(2x+3)^{-3}$

$y \ln 3 = -3\ln(2x+3)$

$\ln 3 \frac{dy}{dx} = \frac{-3}{2x+3}(2)$

$\frac{dy}{dx} = \frac{-6}{(2x+3)\ln 3}$

OR $\frac{dy}{dx} = -3\left(\frac{1}{(2x+3)\ln 3}\right)(2)$

$= \frac{-6}{(2x+3)\ln 3}$

↑ using "result" from lesson

h) $y = 10^{2-3x}$

$\ln y = \ln 10^{2-3x}$

$\ln y = (2-3x)\ln 10$

$\frac{1}{y}\frac{dy}{dx} = -3\ln 10$

$\frac{dy}{dx} = y(-3\ln 10)$

$\frac{dy}{dx} = 10^{2-3x}(-3\ln 10)$

OR $\frac{dy}{dx} = 10^{2-3x}\ln 10(-3)$

$= 10^{2-3x}(-3\ln 10)$

↑ using ar "result"

4. a) $1200(1.12)^5$
$= \$2114.81$

b) $A(t) = 1200(1.12)^t$

c) $A'(t) = 1200(1.12)^t \ln 1.12$

d) $A'(5) = 1200(1.12)^5 \ln 1.12$
$\doteq \$239.67$ per year.

7. $y = 3(2^x)$

$\frac{dy}{dx} = 3(2^x)\ln 2$

at $x=3$
$y = 3(2)^3$
$y = 24$

tangent
$24\ln 2 = \frac{y-24}{x-3}$

$(24\ln 2)(x-3) = y-24$

$(24\ln 2)x - y + 24 - 72\ln 2 = 0$

at $x=3$ $\frac{dy}{dx} = 3(2^3)\ln 2$
$= 24\ln 2 \doteq 16.63$

9. $P(t) = 100(1.2)^{-t}$

a) find t s.t. $P(t) = 50\%$ (originally amount is 100%)

$50 = 100(1.2)^{-t}$
$0.5 = 1.2^{-t}$
$\ln 0.5 = \ln 1.2^{-t}$
$\ln 0.5 = -t \ln 1.2$
$t = -\frac{\ln 0.5}{\ln 1.2}$
$t \doteq 3.8$ years

b) $P'(t) = 100(1.2)^{-t}\ln 1.2(-1)$
$P'(t) = -100(1.2)^{-t}\ln 1.2$
$P'(3.8) \doteq -9.12\%$ per year.

10.

$y = \frac{2x^2}{e^x} = 2x^2e^{-x}$ $\quad \frac{dy}{dx} = 4xe^{-x} + 2x^2(e^{-x})(-1)$

$= 4xe^{-x} - 2x^2e^{-x}$

$(0,0)$ intercept.

C.P. at

$0 = 4xe^{-x} - 2x^2e^{-x}$

$0 = 2xe^{-x}(2-x)$

$x = 0, \quad x = 2$

$y = \frac{8}{e^2}$

C.P. at $(0,0)$ and $(2, \frac{8}{e^2})$

$\frac{d^2y}{dx^2} = 4e^{-x} - 4xe^{-x} - 4xe^{-x} + 2x^2e^{-x}$

$= 4e^{-x} - 8xe^{-x} + 2x^2e^{-x}$

$0 = 2e^{-x}(2 - 4x + x^2)$

$x^2 - 4x + 2 = 0$

$x = 2 \pm \sqrt{2}$ (quad. formula)

$\therefore$ inflection points $(x \doteq 3.41, 0.59)$

$y \doteq 0.77$ or $y \doteq 0.39$

C.P. are turning points

at $x=0$ $\frac{d^2y}{dx^2} = 4e^0 - 8(0) + 12(0)$

$= 4$

> 0

∴ $(0,0)$ a local min

at $x=2$ $\frac{d^2y}{dx^2} = \frac{4}{e^2} - \frac{16}{e^2} + \frac{8}{e^2}$

$= \frac{-4}{e^2} < 0$

∴ $(2, \frac{8}{e^2})$ a local max