

A Review of some Basic Factoring Strategies

x^2

Today we will look at some (not all) of the factoring strategies from the grade 10 math courses. Remember that factoring is the exact opposite of expanding.

Factoring: Finding the Greatest Common Factor

Expand (simplify) each of the following.

a) $2x(x-6)$
 $= 2x^2 - 12x$

b) $-3x(2x+5)$
 $= -6x^2 - 15x$

↑ factoring
↓ expand

Factor each of the following, by identifying the **greatest common factor**.

a) $3x^2 + 9x$
 $= 3x(x+3)$

b) $4y^2 - 16y$
 $= 4y(y-4)$

c) $5f^2 - 3f$
 $= f(5f-3)$

FOIL

Factoring: Simple Trinomials

Expand each of the following. Look for a pattern!

a) $(x+4)(x+3)$
 $= x^2 + 3x + 4x + 12$
 $= x^2 + 7x + 12$

b) $(y+2)(y+3)$
 $= y^2 + 3y + 2y + 6$
 $= y^2 + 5y + 6$

c) $(x-3)(x+2)$
 $= x^2 - x - 6$

d) $(x-4)(x-6)$
 $= x^2 - 10x + 24$

Did you see the pattern? The above are called **simple trinomials**. They have the form $x^2 + bx + c$

$$\begin{array}{c} \uparrow \quad \uparrow \\ x^2 + bx + c \\ + \quad + \end{array}$$

Use your "pattern" to factor each of the following.

a) $x^2 + 9x + 20$
 $= (x+5)(x+4)$

b) $x^2 - 3x - 15$
 $= \text{does not factor}$

c) $x^2 - x - 30$
 $= (x-6)(x+5)$

$$\begin{array}{l} -x_- = -15 \\ -+_- = -3 \end{array}$$

$$\begin{array}{l} 10 \ 6 \ 30 \ 15 \\ 3 \ 5 \ 1 \ 2 \\ -x_- = -30 \\ -+_- = -1 \end{array}$$

d) $s^2 - 5s + 6$
 $= (s-3)(s-2)$

$$\begin{array}{l} \frac{-3}{-3} \ x \frac{-2}{-2} = 6 \\ \frac{-3}{-3} \ + \ \frac{-2}{-2} = -5 \end{array}$$

e) $e^2 - 8e + 12$
 $= (e-6)(e-2)$

You may also remember how to factor trinomials such as the following:
 (but its ok if you don't)

general trinomial

$2x^2 + 5x - 12$

$$= (2x-3)(x+4)$$

Solving Quadratic Equations

Remember that we can solve a linear equation by *isolating the variable*.

$$2x - 5 = 13$$

$$2x = 18$$

$$x = 9$$

This strategy sometimes works for quadratic equations as well:

$$x^2 - 11 = 14$$

$$x^2 = 25$$

$$x = \pm\sqrt{25}$$

$$x = 5 \text{ or } x = -5$$

$$x = \pm 5$$

$$2x^2 - 4 = 16$$

$$\frac{2x^2}{2} = \frac{20}{2}$$

$$x^2 = 10$$

$$x = \pm\sqrt{10}$$

$$x = \pm 3.16$$

However sometimes it does not:

$$x^2 - 5x + 6 = 0$$

~~$$x^2 = 5x - 6$$~~

~~$$x = \sqrt{5x - 6}$$~~

$$\boxed{?} \times \boxed{?} = 0$$

$$\underline{(x-3)} \underline{(x-2)} = 0$$

$$x-3=0 \quad \text{or} \quad x-2=0$$

$$x=3 \quad \text{or} \quad x=2$$

Solve the following by factoring.

$$x^2 + 8x + 15 = 0$$

$$(x+5)(x+3) = 0$$

$$x+5=0 \text{ or } x+3=0$$

$$\boxed{x = -5} \text{ or } \boxed{x = -3}$$

$$x^2 - x = 12$$

$$x^2 - x - 12 = 0$$

$$(x-4)(x+3) = 0$$

$$x-4=0 \text{ or } x+3=0$$

$$\boxed{x = 4} \text{ or } \boxed{x = -3}$$

$$4n^2 + 8n = 0$$

$$4n(n+2) = 0$$

$$4n = 0 \text{ or } n+2 = 0$$

$$n = 0 \quad n = -2$$

But what if a quadratic equation cannot be factored?

Solve $x^2 + 9x - 1 = 0$

$-x - = -1$ $-1, 1$
 $- + - = 9$

In this case we can use the quadratic formula.

If $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x^2 + 9x - 1 = 0$
 $a=1$ $b=9$ $c=-1$
 $x = \frac{-9 \pm \sqrt{9^2 - 4(1)(-1)}}{2(1)}$
 $x = \frac{-9 \pm \sqrt{85}}{2}$
 $x \approx 0.11$ $x \approx -9.11$

Solve using the quadratic formula:

$x = 1 - 6x^2$

$6x^2 + 1x - 1 = 0$
 a b c

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-1 \pm \sqrt{1^2 - 4(6)(-1)}}{2(6)}$

$x = \frac{-1 \pm \sqrt{25}}{12}$

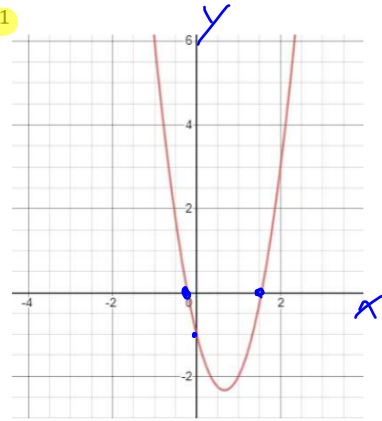
$x = \frac{-1 \pm 5}{12}$

$x = \frac{4}{12}$ or $x = \frac{-6}{12}$
 $x = \frac{1}{3}$ $x = -\frac{1}{2}$

$(3x-1)(2x+1) = 0$
 $3x-1=0$ or $2x+1=0$
 $3x=1$ or $2x=-1$
 $x = \frac{1}{3}$ $x = -\frac{1}{2}$

Find the x and y-intercepts for the graph of $y = 3x^2 - 4x - 1$

y-int $x = 0$
 $y = 3(0)^2 - 4(0) - 1$
 $y = -1 \quad (0, -1)$



x-int $y = 0$

$$0 = 3x^2 - 4x - 1 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{4 \pm \sqrt{28}}{6} \rightarrow x \doteq 1.55$$

$$x \doteq -0.215$$

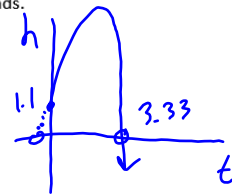
A model rocket is launched upwards with a velocity of 16 m/s. The equation that models the height of the rocket is $h(t) = -4.9t^2 + 16t + 1.1$ where $h(t)$ is the height (in metres) after t seconds.

a) Find the initial height of the rocket. $t = 0$

$$h(0) = -4.9(0)^2 + 16(0) + 1.1$$

$$h(0) = 1.1$$

1.1m



b) How long is the rocket in the air for? (round to 2 decimal places)

$$h(t) = 0$$

$$0 = -4.9t^2 + 16t + 1.1$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-16 \pm \sqrt{16^2 - 4(-4.9)(1.1)}}{2(-4.9)}$$

$$t = \frac{-16 \pm \sqrt{277.56}}{-9.8}$$

3.33
seconds

$$t \doteq -0.067 \quad \text{or} \quad \boxed{t \doteq 3.33}$$

$t > 0$

